

Math 2250–1

Week 13 concepts and homework, due November 23.

Recall that problems which are underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

*7.1: modeling coupled mass–spring systems or multi–component input–output systems with systems of differential equations; converting single differential equations or systems of differential equations into equivalent first order systems of differential equations by introducing functions for the intermediate derivatives; comparing solutions to these equivalent systems.*

7.1: 1, 2, 5, 9, 13, 21, 22a, 24, 26.

**w13.1)** This is a continuation of 22a.

**a)** Use the chain rule to show directly that solutions  $x(t), y(t)$  of the system

$$\begin{aligned}x'(t) &= -2y(t) \\ y'(t) &= 2x(t)\end{aligned}$$

satisfy the equation

$$x^2(t) + y^2(t) = C^2.$$

**b)** Use pplane (google it on your browser) to draw the phase portrait for the first order system in this problem. Use the option which lets you plot solutions to initial value problems in order to sketch the parametric curve  $[x(t), y(t)]^T$  for the solution to the initial value problem for this system, with  $x(0) = 1$  and  $y(0) = 0$ . Print out a screen shot of your work to hand in. (Note that  $x(t)$  satisfies  $x''(t) + x(t) = 0$ , so the phase portrait is plotting the position and velocity points for simple harmonic motion.)

*7.2) recognizing homogeneous and non–homogeneous linear systems of first order differential equations; writing these systems in vector–matrix form; statement of existence and uniqueness for IVP's in first order systems of DE's and its consequences for the dimension of the solution space to the first order system, and for the general solution  $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_H$  to the non–homogeneous system. Using the*

*Wronskian to check for bases.*

7.2: 1, 9, 12, 13, 14, 23.

**w13.2)** This is a continuation of 14, 23:

**a)** Use the eigenvalue–eigenvector method of section 7.3 to generate a basis for the general solution in 14. It should be consistent with the basis you were given in this problem.

**b)** Use pplane to draw the phase portrait for this first order system, along with the parametric curve of the solution  $[x(t), y(t)]^T$  to the initial value problem in 23. Print out a screen shot of your work to hand in.

**w13.3) This problem is postponed until the week after Thanksgiving.** Now consider the non–homogeneous system

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

**a)** Use the method of undetermined coefficients to find a particular solution of the form

$$\mathbf{x}_p = t \cdot \mathbf{a} + \mathbf{b}$$

**b)** Use the Laplace transform (transform each of the two differential equations), to find a particular solution with  $x(0) = 0 = y(0)$ . Notice you will arrive at a matrix system for  $[X(s), Y(s)]^T$  that you can solve using the adjoint formula for inverses of  $2 \times 2$  matrices. Once you find  $X(s), Y(s)$ , you may use

Maple or other technology to compute the partial fractions decomposition of  $X(s)$  and  $Y(s)$ , in order to recover  $x(t)$ ,  $y(t)$ .

c) Check your work with the Maple "dsolve" command, if you wish.

7.3) *the eigenvalue–eigenvector method for finding the solution space to homogeneous constant coefficient first order systems of differential equations: real and complex eigenvalues.*

7.3: 3, 13, 29, 31, **32**, 36.

**w13.4)** Use the eigenvalue–eigenvector method to solve the initial value problem for the two coupled tanks discussed on pages 1 and 5 of our Friday Nov. 18 class notes. Your solution should be consistent with the picture on page 5 of those notes.

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \end{bmatrix}$$

**w13.5)** Use the eigenvalue–eigenvector method (with complex eigenvalues) to solve the first order system initial value problem which is equivalent to the second order differential equation IVP on page 4 of Friday Nov. 18 notes. Your solution to the first order system should correspond to the solution to this underdamped unforced oscillator IVP that we found on Friday. (Also note that the Wronskian matrix for the first order system is exactly the Wronskian matrix from Chapter 5, for the second order differential equation.)

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -17 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$
$$\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$