Math 2250–1 Week 10 concepts and homework, due November 4.

Recall that problems which are underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

5.5: finding particular solutions using the method of undetermined coefficients; using variation of parameters; using the general solution $y = y_p + y_H$ to solve associated initial value problems. 5.5: **2**, 3, **10**, 21, 27, 29, **32**, **34**, 43, **45**, 47, **52**, 57, **58**.

5.6: solving forced oscillation problems; understanding resonance in undamped problems, steady periodic and transient solutions in damped problems, and practical resonance in slightly damped problems; mass-spring applications; finding natural frequencies in more general (undamped) conservative systems via conservation of energy equations.

5.6: 3, 5, 7, 8, 11, 12, 17, 18, 21, 22. Note: the answer to #12 in the back of the book has a typo – the cosine in the steady-periodic solution should be evaluated at $5 \cdot t - \alpha$ rather than at $3 \cdot t - \alpha$.

EP3.7: understanding unforced and forced oscillation problems for RLC circuits: EP3.7: 11, <u>12, 17, 20</u>.

Notes: You may use formulas we've derived in class or that the text derives, for particular solutions to the forced oscillation problems of sections 5.6 and EP3.7. I'd recommend that you work some of these problems using the method of undetermined coefficients, however, because the practice will help you solidify your mathematical understanding.

Also, these problems can get somewhat computational. If you wish to check your work, you can always use Maple. For example, for 5.6.12 you can't just plug into the solution formula we worked out in class and that the book displays as formulas (21),(22) on page 359 – since the forcing term is a multiple of $\sin(\omega \cdot t)$ rather than $\cos(\omega \cdot t)$. However, you can easily check your intermediate work (before converting to amplitude–phase form), as follows:

> with(DEtools):
> deqtn := diff(x(t), t, t) + 6 · diff(x(t), t) + 13 · x(t) = 10 · sin(5 · t);
ics := x(0) = 0, D(x)(0) = 0;
deqtn :=
$$\frac{d^2}{dt^2} x(t) + 6 \left(\frac{d}{dt} x(t)\right) + 13 x(t) = 10 sin(5 t)$$

ics := x(0) = 0, D(x)(0) = 0 (1)
> dsolve({deqtn, ics}, x(t));
x(t) = $\frac{125}{174} e^{-3t} sin(2 t) + \frac{25}{87} e^{-3t} cos(2 t) - \frac{10}{87} sin(5 t) - \frac{25}{87} cos(5 t)$ (2)

(in practice, decimal answers are cleaner to write down, but this way you can check against the solution at the back of the book.)