Review Sheet

Math 2250-1, November 2011

Our exam covers chapters 4.1-4.4, 5, EP3.7, and 10.1-10.3 of the text. Only scientific calculators will be allowed on the exam.

8:30-9:30 JFB 103 Thursday.

Chapter 4.1-4.4

At most 30% of the exam will deal directly with this material....but much of Chapter 5 uses these concepts, so much more than 30% of the exam will be related to chapter 4. (And, as far as matrix and determinant computations go, you should remember everything you learned in Chapter 3.)

Do you know the key definitions?

vector space

any space in which you can scalar multiply and add the object in the space, so that you stay in the space and so that the usual rules for scalar multiplication and addition hold...with the usual rules for algebra for add and scalar multiply.....some texts call these "linear combination spaces".

a linear combination of a collection $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$ of k vectors

is any sum of scalar multiples, i.e. any $\underline{\boldsymbol{v}} = c_1\underline{\boldsymbol{v}}_1 + \ldots.\ c_k\underline{\boldsymbol{v}}_k$.

linearly independent vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$

If $c_1 v_1 + \ldots. c_k v_k = 0$ *then all the* c_j*'s* $= 0$.

linearly dependent vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$

not independent, i.e. linear combos do add up to zero, when not all the c_j*'s* $= 0$. *Another way to say this is that at least one of the vectors is a linear combination of (some of) the others.*

span of vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$

the collection of all linear combinations of $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$.

subspace of a vector space

any subset that is closed under addition and scalar multiplication, i.e. whenever $\underline{\boldsymbol{v}}$, $\underline{\boldsymbol{w}}$ are in the subspace, then so are $\underline{\boldsymbol{v}} + \underline{\boldsymbol{w}}$ and $c \cdot \underline{\boldsymbol{v}}$ (if c is any scalar)....*another way to say this is that a subspace is a subset that is closed with respect to linear combinations, since any linear combination is constructed by a sequence of sums and scalar multiple. Since the subspace is in the larger vector space, the rules of arithmetic also hold in the subspace, and it is itself a vector space.*

basis of a vector space

a collection of vectors that is independent, and that spans the space

how do you get a basis if you already have a (possibly dependent) spanning set?

delete dependent ones (ones that are linear combos of the remaining vectors)...until you have an independent spanning set

how do you get a basis if you have an independent set that doesn't span the vector space?

augment with vectors not in the span of what you've got so far, until you add enough vectors to get a basis.

dimension of a vector space
the number of vectors in any basis.

Subspace examples from Chapter 4, involving the concepts above

(1) solutions to matrix equation $[A]\underline{\mathbf{x}} = \underline{\mathbf{0}}$, because if $[A]\underline{\mathbf{x}} = \underline{\mathbf{0}}$ *and* $[A]\underline{\mathbf{y}} = \underline{\mathbf{0}}$ *then* $[A](\underline{\mathbf{x}} + \underline{\mathbf{y}}) = [A]\underline{\mathbf{x}} + [A]\underline{\mathbf{y}} = \underline{0} + \underline{0} = \underline{0}$ *and* $[A]c\underline{\mathbf{x}} = c[A]\underline{\mathbf{x}} = \underline{\mathbf{0}}$.

How to find general solution? *ans: backsolve from rref.* basis? *write the general solution in linear combination form - the vectors you used will be a basis: By construction they span; by setting a linear combination equal to zero you can look at the various entries of the linear combination to deduce that the linear combination coefficients are all zero.*

(2) span of vectors $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_k$, *because if you add two linear combinations of* $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_k$ *and use arithmetic rules to collect terms, you'll still have a linear combination. And if you multiply a linear combination by a constant, it will still be a linear combination.*

Subspace examples from Chapter 5

(3) solutions to homogeneous linear differential equation for e.g. $y = y(x)$ on an interval I, i.e. solutions to

$$L(y) := y^{(n)} + p_{n-1}(x)\cdot y^{(n-1)} + \ldots + p_1(x)\cdot y' + p_0(x)\cdot y = 0$$

because if $L(y_1) = 0$ *and* $L(y_2) = 0$, *then* $L(y_1 + y_2) = L(y_1) + L(y_2) = 0 + 0 = 0$, *and* $L(c\cdot y_1) = c(L(y_1)) = c\cdot 0 = 0$.

What is the general solution to L(y)=f, if L is a linear operator? (What does it mean for an operator or transformation to be linear?)

$y = y_p + y_H$ *where* y_P *is any particular solution, and* y_H *is the general homogeneous solution.*

L is linear means it's always true that $L(y_1 + y_2) = L(y_1) + L(y_2)$ ***and*** $L(c\cdot y_1) = c\cdot L(y_1)$.
In physics they call this superposition.

Examples?

solution space to $[A]\underline{\mathbf{x}} = \underline{\boldsymbol{b}}$

solution space to $L(y) = f$, i.e. the non-homogeneous linear DE.

Chapter 5 and EP3.7 (circuits).
At least 60% of the exam will be related to this material.

What is the **natural initial value problem** for n^{th}-order linear differential equation, i.e. the one that has unique solutions?

$$L(y) := y^{(n)} + p_{n-1}(x)\cdot y^{(n-1)} + \ldots + p_1(x)\cdot y' + p_0(x)\cdot y = f(x)$$
$$y(x_0) = b_0$$
$$y'(x_0) = b_1$$
$$y^{(n-1)}(x_0) = b_{n-1}.$$

What is the **dimension of the solution space to the homogeneous DE**

$$L(y) := y^{(n)} + p_{n-1}(x)\cdot y^{(n-1)} + \ldots + p_1(x)\cdot y' + p_0(x)\cdot y = 0 \ \ ?$$
$$n$$

How can you tell if $y_1(x), y_2(x), \ldots y_n(x)$ are a **basis** for the homogeneous solution space above?

If you try to match $c_1 y_1(x) + c_2 y_2(x) + \ldots c_n \cdot y_n(x)$ to the initial values....at $x = x_0$ you get a matrix system which is the Wronskian matrix at $x = x_0$ times the **c** *vector, equals the* **b** *vector. If that matrix is invertible you can always find a unique* **c** *vector to solve the initial value problem. (And you can check this by taking the Wronskian determinant at $x = x_0$.) Since any solution satisfies some intial value problem and because there's only one solution for each initial value problem, this shows $y_1(x), y_2(x), \ldots y_n(x)$ span the solution space.*

If a linear combination of $y_1(x), y_2(x), \ldots y_n(x)$ added up to the zero function, which has a **b** *vector which is all zero's, then the* **c** *vector would also have to be all zeroes, which verifies independence.*

How is your answer above related to a **Wronskian matrix** and the **Wronskian determinant**?

How do you find the **general solution** to the **homogeneous constant coefficient linear DE**

$$L(y) := a_n \cdot y^{(n)} + a_{n-1}\cdot y^{(n-1)} + \ldots + a_1 \cdot y' + a_0 \cdot y = 0\ ?$$

(your answer should involve the characteristic polynomial, Euler's formula, repeated roots, complex roots.)

search for exponential (or related solutions), using roots of the characteristic polynomial. Know the special cases above.

What is another word for the "**principle of superposition**"?

linearity: If $L(y_1) = f_1$ and $L(y_2) = f_2$ then $L(c_1 \cdot y_1 + c_2 \cdot y_2) = c_1 \cdot f_1 + c_2 \cdot f_2$.

What form does the **general solution to the non-homongeneous linear differential equation**

$$L(y) := y^{(n)} + p_{n-1}(x)\cdot y^{(n-1)} + \ldots + p_1(x)\cdot y' + p_0(x)\cdot y = f(x)$$

take?

$$y = y_P + y_H.$$

What three (?!) ways do you know to find **particular solutions to constant coefficient non-homogeneous linear DEs**

$$L(y) := a_n \cdot y^{(n)} + a_{n-1} \cdot y^{(n-1)} + \ldots + a_1 \cdot y' + a_0 \cdot y = f?$$

undetermined coefficients....educated guessing...know the rules.
variation of parameters. (not on the test)
Laplace transform.

5.4, 5.6, EP3.7 Mechanical vibrations and forced oscillations; electrical circuit analog

What are the governing second order DE's for a **damped mass-spring configuration** (Newton's second law) and for an **RLC circuit (**Kirchoff's Law for potential energy drop)?

$m \cdot x''(t) + c \cdot x'(t) + k \cdot x(t) = F(t)$

$L \cdot Q''(t) + R \cdot Q'(t) + \frac{1}{C} \cdot Q(t) = E(t)$

What are **unforced undamped oscillations**, and their **solution formulas/behavior**?

$c = 0$, $F = 0$ simple harmonic motion $x(t) = A \cdot \cos(\omega_0 \cdot t) + B \cdot \sin(\omega_0 \cdot t)$ *ω_0 is the square root of* $\frac{k}{m}$.

Can you convert a linear combination $A \cdot \cos(\omega \cdot t) + B \cdot \sin(\omega \cdot t)$ in amplitude-phase form? Can you explain the physical properties of the solution?

be able to do this...amplitude, phase, angular frequency, frequency, period, time delay

What are **unforced damped oscillations**, and their **solution formulas/behavior (three types)?**

$c$is not zero, $F = 0$ .... *under*, *over*, *critical damping ...you can tell by looking at the roots of the characteristic polynomial*

What are the possible phenomena with **forced undamped oscillations** (assuming the forcing function is sinusoidal)?

resonance $\omega = \omega_0$ *(know this for the exam), and beating(which will not be on the exam).*

What are the possible phenomena with **forced damped oscillations** (assuming the forcing function is sinusoidal)?

practical resonance (not on exam), *also steady periodic and transient parts of the solution (know this.)*

Can you solve all initial value problems that arise in the situations above?

hopefully.

Can you use **total energy in conservative systems, TE=KE+PE,** to derive the second order DE which is Newton's law, at least for examples we've discussed?

this is a very important concept which you will see in later courses, and which we discussed. But it's unlikely to be on this midterm exam.

Chapter 10: Laplace transform techniques

At least 25% of the exam will deal directly with this material. The most efficient way for me to test it is to have you solve IVPs from chapter 5 (or earlier), using chapter 5 techniques as well as Laplace transform techniques. You will be responsible for 10.1-10.3 on the exam. (Sections 10.4-10.5 are yet to come.) You will be provided with the Laplace transform table on the front book cover.

Can you use the **definition of Laplace transform** to compute Laplace transforms?

Can you **convert a constant coefficient linear differential equation IVP** into an algebraic expression for the **Laplace transform $X(s)$ of the solution $x(t)$**?

Can you use **partial fraction techniques** and **completing the square algebra** to break $X(s)$ into a linear combination of simpler functions for which the Laplace transform table will enable you to compute $\mathcal{L}^{-1}\{X(s)\}(t)$?