Review Sheet
Math 2250−1, November 2011

Our exam covers chapters 4, 5, EP3.7, and 10.1−10.3 of the text. Only scientific calculators will be allowed on the exam.

Chapter 4.1−4.4
At most 30% of the exam will deal directly with this material....but much of Chapter 5 uses these concepts, so much more than 30% of the exam will be related to chapter 4. (And, as far as matrix and determinant computations go, you should remember everything you learned in Chapter 3.)
Do you know the key definitions?

vector space

linear combination of a collection $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$ of k vectors

linearly independent vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$

linearly dependent vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$

span of vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$

subspace of a vector space

basis of a vector space
how do you get a basis if you already have a (possibly dependent) spanning set?
how do you get a basis if you have an independent set that doesn't span the vector space?

dimension of a vector space

Subspace examples from Chapter 4, involving the concepts above
solutions to matrix equation $[\boldsymbol{A}]\underline{\mathbf{x}} = \underline{\mathbf{0}}$
span of vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_k$

Subspace examples from Chapter 5
solutions to homogeneous linear differential equation for e.g. $y = y(x)$ on an interval I, i.e. solutions to

$$L(y) := y^{(n)} + p_{n-1}(x) \cdot y^{(n-1)} + \ldots + p_1(x) \cdot y' + p_0(x) \cdot y = 0$$

What is the general solution to L(y)=f, if L is a linear operator? (What does it mean for an operator or transformation to be linear?)

Examples?
solution space to $[\boldsymbol{A}]\underline{\boldsymbol{x}} = \underline{\boldsymbol{b}}$
solution space to $L(y) = f$, i.e. the non−homogeneous linear DE.

Chapter 5 and EP3.7 (circuits).

At least 50% of the exam will be related to this material.

What is the **natural initial value problem** for n^{th} – order linear differential equation, i.e. the one that has unique solutions?

$$L(y) := y^{(n)} + p_{n-1}(x)\cdot y^{(n-1)} + \ldots + p_1(x)\cdot y' + p_0(x)\cdot y = f(x)$$

What is the **dimension of the solution space to the homogeneous DE**

$$L(y) := y^{(n)} + p_{n-1}(x)\cdot y^{(n-1)} + \ldots + p_1(x)\cdot y' + p_0(x)\cdot y = 0 \ ?$$

How can you tell if $y_1(x), y_2(x), \ldots y_n(x)$ are a **basis** for the homogeneous solution space above?

How is your answer above related to a **Wronskian matrix** and the **Wronskian determinant**?

How do you find the **general solution** to the **homogeneous constant coefficient linear DE**

$$L(y) := a_n\cdot y^{(n)} + a_{n-1}\cdot y^{(n-1)} + \ldots + a_1\cdot y' + a_0\cdot y = 0 \ ?$$

(your answer should involve the characteristic polynomial, Euler's formula, repeated roots, complex roots.)

What is another word for the "**principle of superposition**"?

What form does the **general solution to the non−homongeneous linear differential equation**

$$L(y) := y^{(n)} + p_{n-1}(x) \cdot y^{(n-1)} + \ldots + p_1(x) \cdot y' + p_0(x) \cdot y = f(x)$$

take?

What three (?!) ways do you know to find **particular solutions to constant coefficient non−homogeneous linear DEs**

$$L(y) := a_n \cdot y^{(n)} + a_{n-1} \cdot y^{(n-1)} + \ldots + a_1 \cdot y' + a_0 \cdot y = f\,?$$

5.4, 5.6, EP3.7 Mechanical vibrations and forced oscillations; electrical circuit analog

What are the governing second order DE's for a **damped mass−spring configuration** (Newton's second law) and for an **RLC circuit (**Kirchoff's Law for potential energy drop)?

What are **unforced undamped oscillations**, and their **solution formulas/behavior**?

Can you convert a linear combination $A \cdot \cos(\omega \cdot t) + B \cdot \sin(\omega \cdot t)$ in amplitude−phase form? Can you explain the physical properties of the solution?

What are **unforced damped oscillations**, and their **solution formulas/behavior (three types)?**

What are the possible phenomena with **forced undamped oscillations** (assuming the forcing function is sinusoidal)?

What are the possible phenomena with **forced damped oscillations** (assuming the forcing function is sinusoidal)?

Can you solve all initial value problems that arise in the situations above?

Can you use **total energy in conservative systems, TE=KE+PE,** to derive the second order DE which is Newton's law, at least for examples we've discussed?

Chapter 10: Laplace transform techniques

At least 25% of the exam will deal directly with this material. The most efficient way for me to test it is to have you solve IVPs from chapter 5 (or earlier), using chapter 5 techniques as well as Laplace transform techniques. You will be responsible for 10.1−10.3 on the exam. (Sections 10.4−10.5 are yet to come.) You will be provided with the Laplace transform table on the front book cover.

Can you use the **definition of Laplace transform** to compute Laplace transforms?

Can you **convert a constant coefficient linear differential equation IVP** into an algebraic expression for the **Laplace transform $\boldsymbol{X(s)}$ of the solution $\boldsymbol{x(t)}$**?

Can you use **partial fraction techniques** and **completing the square algebra** to break $X(s)$ into a linear combination of simpler functions for which the Laplace transform table will enable you to compute $\mathcal{L}^{-1}\{X(s)\}(t)$?