

Fri 12/9

Review day

- Final exam is next Friday, 12/16 8-10:00 a.m. (You will be able to work until 10:20.)
- Chance to go over last spring's final exam next week. I suggest Wed 12/14 3-5 p.m.?

Chapters 1-2	10-20%	1 <sup>st</sup> order DE's	
3-4	20-30%	matrix algebra and vector space concepts	
5	15-30%	linear DE's	
6	15-30%	eigenvalues, eigenvectors	} new since 2 <sup>nd</sup> midterm.
7	20-40%	linear systems of DE's	
9	15-25%	non-linear systems of DE's	
10	15-25%	Laplace transform	

(topics overlap, so percentages add up to more than 100%)

We'll discuss how the topics below are interconnected.

1-2 1<sup>st</sup> order DE's.

slope fields, phase portraits (for autonomous DE's)  
 equil. sol'ns  
 stability  
 ∃! for IVP  
 methods:  
 separable  
 linear  
 applications  
 populations  
 vel-accel. models  
 tanks

3-4 Matrix alg/vector spaces

linear systems & matrices  
 rref  
 matrix algebra, manipulating matrix-vector equations  
 $A^{-1}$   
 $|A|$   
 vector spaces/subspaces  
 linear dep/indep.  
 span  
 basis  
 dimension  
 examples

5. Linear DE's

IVP ∃!  
 constant coeff DE's  
 basis of sol'ns for  $x_H \sim e^{rt}$ , Euler's formula  
 undetermined coeffs for  $x_p$   
 applications to mechanical systems  
 damped/undamped; forced/unforced phenomena  
 amplitude/phase form of sinusoidal fns  
 using  $TE = KE + PE$  for conservative systems

6. Eigenvalues & eigenvectors

A eigenbasis  
 (we won't consider defective matrices for final exam)  
 algebra for real and complex eigenvalues & eigenvectors

7. Linear systems of DE's.

$\vec{x}' = A\vec{x}$   $e^{At}\vec{v}$  bases  
 $\vec{x}' = A\vec{x} + \vec{f}(t)$ ,  $\vec{x} = \vec{x}_p + \vec{x}_H$   
 $\vec{x}'' = A\vec{x}$  if A arises from conservative system  
 $\vec{x}'' = A\vec{x} + \cos\omega t \vec{b}$   $\cos\omega t \vec{v}_1$ ,  $\sin\omega t \vec{v}_2$  bases  
 $\vec{x} = \vec{x}_p + \vec{x}_H$

natural IVP's, ∃!, dim of sol'n space for homog. linear systems.  
 equivalence of any DE or system of DE's to a 1<sup>st</sup> order system, and consequences  
 applications to input/output models & to multi-component mechanical systems

9. Nonlinear systems of DE's

autonomous systs of 2 1<sup>st</sup> order DE's  
 equil. sol'ns  
 stability  
 linearization near equilibria  
 population & mechanical system modelling

10. Laplace transform

def.  
 using table for  $\mathcal{L}$ ,  $\mathcal{L}^{-1}$   
 IVP's for linear DE's or systems via Laplace ~ of fns with partial fractions

We can probably touch on 70% of course concepts by studying these two DE's in as many ways as we can think of.

$$x'' + 5x' + 4x = 0$$

$$x'' + 5x' + 4x = 3 \cos 2t$$

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