Math 2250-1

Wednesday Sept 24

- There's an interesting discussion on page 2 Tuesday about the general form of sol'ns to linear equations, and it ties back into DE's! We'll go over this in class.

Then, §3.4

- Matrix algebra

<u>addition</u>: If A and B are both $m \times n$, then

$$\text{entry}_{ij}(A+B) := a_{ij} + b_{ij}$$

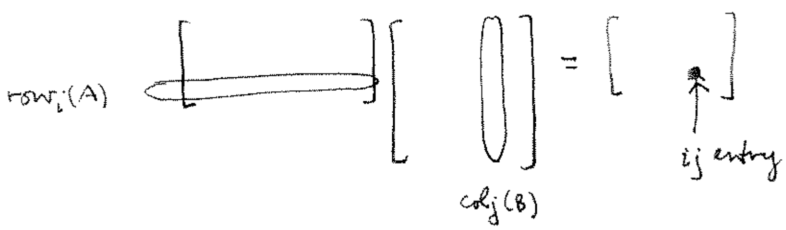
<u>Scalar multiplication</u>:

$$\text{entry}_{ij}(cA) := c a_{ij}$$

just like vector addition and scalar multiplication

<u>matrix multiplication</u>: [generalizes matrix times vector]

$$\text{entry}_{ij}(AB) = \text{row}_i(A) \cdot \text{col}_j(B)$$



so only works for

$$\left[A\right]_{m\times n}\left[B\right]_{n\times p} = \left[AB\right]_{m\times p}$$

<u>examples</u>:

Rules for this algebra

$+$ is commutative $\quad A+B=B+A$

$+$ is associative $\quad (A+B)+C = A+(B+C)$

scalar mult distributes over $+$ $\quad c(A+B) = cA + cB$

mult is associative $\quad A(BC) = (AB)C$

mult distributes over $+$ $\quad A(B+C) = AB+AC$

$$(A+B)C = AC+BC$$

mult <u>not</u> commutative in general : DON'T EXPECT $AB=BA$!

<u>check properties</u> :

matrix algebra continued...

the $(n\times n)$ identity matrix $I := \begin{bmatrix} 1 & & & \\ 0 & 1 & & \bigcirc \\ 0 & 0 & \ddots & \\ & \bigcirc & & 1 \end{bmatrix}$ 　 $\text{entry}_{jj}(I) = 1$
$\text{entry}_{ij}(I) = 0$ if $i \neq j$

$$\text{col}_j(I) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \leftarrow \text{entry } j \quad := \vec{e}_j$$

$$\text{row}_i(I) = [0, 0, \ldots \underset{\substack{\uparrow \\ \text{entry } i}}{1}, 0 \cdots 0] = \vec{e}_i \quad \text{(written as a row vector)}$$

$$A_{m\times n} I_{n\times n} = A$$

check: $\text{entry}_{ij}(A\,I) = \text{row}_i(A) \cdot \text{col}_j(I) = \text{row}_i(A) \cdot \vec{e}_j = a_{ij} = \text{entry}_{ij}(A)$

$$I_{m\times m} A_{m\times n} = A$$

check:

<u>Definition</u> $A_{n\times n}$ is <u>invertible</u> $:=$ there is a matrix B s.t. $AB = BA = I$
(and then we write A^{-1} for B)

[A can have at most 1 inverse, since if
$AB = BA = I$
and $AC = CA = I$
then $B(AC) = (BA)C$ 　 associative prop
$B\ I = I\ C$
$B = C$!]

<u>Example</u> If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, show $A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$

we'll see why inverse matrices are important, and how to find them (when they exist), on Friday!!