

Math 2250-1
Tuesday Sept 23

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We're going through the Maple examples from Monday, but first:

We introduced some useful notation yesterday which should be recorded (and expanded):

linear system of m eqns in n unknowns:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + \dots & \\ \vdots & \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

rewritten in matrix form:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

i.e.

$$A \vec{x} = \vec{b}$$

A = coefficient matrix
 \vec{b} = RHS vector

exercise: $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} =$

important properties of matrix times vector:

entry i : $(A\vec{x})_i = \text{row}_i(A) \cdot \vec{x}$ (dot product)

recall, if $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$, $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

$$\begin{aligned} \vec{u} \cdot [\vec{v} + \vec{w}] &= \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w} \\ \vec{u} \cdot [k\vec{v}] &= k\vec{u} \cdot \vec{v} \end{aligned}$$

then $\vec{u} \cdot \vec{v}$ equals $u_1v_1 + u_2v_2 + \dots + u_nv_n$
 $= \sum_{i=1}^n u_i v_i$

(this is the def. of dot product!!)

$$\text{So } A(\vec{x} + \vec{y}) = A\vec{x} + A\vec{y}$$

$$A(k\vec{x}) = kA\vec{x}$$

Check:

now
~~then~~ finish Monday Maple handout.

after our general overview/conclusions on page 6 Monday,
let's return to a question we asked on pages 2-3 ...

Theorem The general solution to

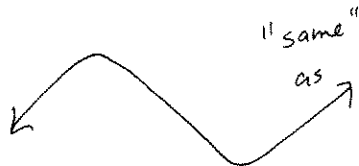
$$A\vec{x} = \vec{b}$$

is of the form $\vec{x} = \vec{x}_p + \vec{x}_H$

↑
(any) particular solution

← general solution to the homogeneous equation $A\vec{x} = \vec{0}$

proof: If $A\vec{x}_p = \vec{b}$ and $A\vec{x}_H = \vec{0}$
then $A(\vec{x}_p + \vec{x}_H) = A\vec{x}_p + A\vec{x}_H$
 $= \vec{b} + \vec{0} = \vec{b}$



"same"
as

(let $L(y) := y' + p(x)y$
 $L(y_1 + y_2) = L(y_1) + L(y_2)$
 $L(cy) = cL(y)$
check!

so same conclusion holds,
to solve $L(y) = q(x)$,

i.e. $y' + p(x)y = q(x)$

gen'l sol'n is $y = y_p + y_H!$

If \vec{x} is any sol'n to
 $A\vec{x} = \vec{b}$

then
 $\vec{x} = \vec{x}_p + (\vec{x} - \vec{x}_p)$

so $A\vec{x} = A[\vec{x}_p + (\vec{x} - \vec{x}_p)]$
 $= A\vec{x}_p + A(\vec{x} - \vec{x}_p)$

so $\vec{b} = \vec{b} + A(\vec{x} - \vec{x}_p)$

so $\vec{0} = A(\vec{x} - \vec{x}_p)$

so $\vec{x} - \vec{x}_p$ is a sol'n to
the homog.
eqn, i.e.

$$\vec{x} - \vec{x}_p = \vec{x}_H$$

example

$$y' + 3y = 6$$

$$e^{3x}(y' + 3y) = 6e^{3x}$$

$$(e^{3x}y)' = 6e^{3x}$$

$$e^{3x}y = 2e^{3x} + C$$

$$y = 2 + \underbrace{C e^{-3x}}$$

↑ ↑
 y_p y_H

!!