Math 2250-1
Tuesday Sept 23

We're going through the Maple examples from Monday, but first:
We introduced some useful notation yesterday which should be recorded (and expanded):

linear system of m eqtns in n unknowns:

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + \quad a_{1n}x_n &= b_1\\
a_{21}x_1 + a_{22}x_2 + \cdots + \quad a_{2n}x_n &= b_2\\
a_{31}x_1 + \cdots\\
\vdots\\
a_{m1}x_1 + a_{m2}x_2 + \cdots \quad + a_{mn}x_n &= b_m
\end{aligned}$$

rewritten in matrix form:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \vdots \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn}\end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n\end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m\end{bmatrix}$$

i.e. $A\vec{x} = \vec{b}$ $\qquad$ A = coefficient matrix, $\vec{b}$ = RHS vector

<u>exercise</u>: $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 4\end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ 3\end{bmatrix} =$

important properties of matrix times vector:

$\text{entry}_i(A\vec{x}) = \text{row}_i(A)\cdot\vec{x}$ (dot product)

recall, if $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n\end{bmatrix}$, $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n\end{bmatrix}$ then $\vec{u}\cdot\vec{v}$ equals $u_1v_1 + u_2v_2 + \dots + u_nv_n = \sum_{i=1}^{n} u_iv_i$

(this is the def. of dot product!!)

$\vec{u}\cdot[\vec{v}+\vec{w}] = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$

$\vec{u}\cdot[k\vec{v}] = k\vec{u}\cdot\vec{v}$

So $A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y}$

$A(k\vec{x}) = kA\vec{x}$

<u>Check</u>:

~~then~~ now finish Monday Maple handout.

after our general overview/conclusions on page 6 Monday,
let's return to a question we asked on pages 2-3 ...

<u>Theorem</u> The general solution to

$$A\vec{x} = \vec{b}$$

is of the form $\vec{x} = \vec{x}_p + \vec{x}_H$

(any) particular solution

general solution to the homogeneous equation $A\vec{x} = \vec{0}$

proof: If $A\vec{x}_p = \vec{b}$ and $A\vec{x}_H = \vec{0}$
then $A(\vec{x}_p + \vec{x}_H) = A\vec{x}_p + A\vec{x}_H$
$= \vec{b} + \vec{0} = \vec{b}$

"same" as

let $L(y) := y' + p(x)y$
$L(y_1 + y_2) = L(y_1) + L(y_2)$
$L(cy) = cL(y)$
Check!

So same conclusion holds, to solve $L(y) = q(x)$,
i.e. $y' + p(x)y = q(x)$
gen'l sol'n is $y = y_p + y_H$!

If $\vec{x}$ is <u>any</u> soln to
$A\vec{x} = \vec{b}$
then
$\vec{x} = \vec{x}_p + (\vec{x} - \vec{x}_p)$
so $A\vec{x} = A[\vec{x}_p + (\vec{x} - \vec{x}_p)]$
$= A\vec{x}_p + A(\vec{x} - \vec{x}_p)$
so $\vec{b} = \vec{b} + A(\vec{x} - \vec{x}_p)$
so $\vec{0} = A(\vec{x} - \vec{x}_p)$
so $\vec{x} - \vec{x}_p$ <u>is</u> a sol'n to the homog. eqtn, i.e.
$\vec{x} - \vec{x}_p = \vec{x}_H$

∎

example

$$y' + 3y = 6$$
$$e^{3x}(y' + 3y) = 6e^{3x}$$
$$(e^{3x}y)' = 6e^{3x}$$
$$e^{3x}y = 2e^{3x} + C$$
$$y = \underset{y_P}{2} + \underset{y_H}{\underbrace{Ce^{-3x}}}$$

!!