Math 2250-1
Tuesday Sept 2

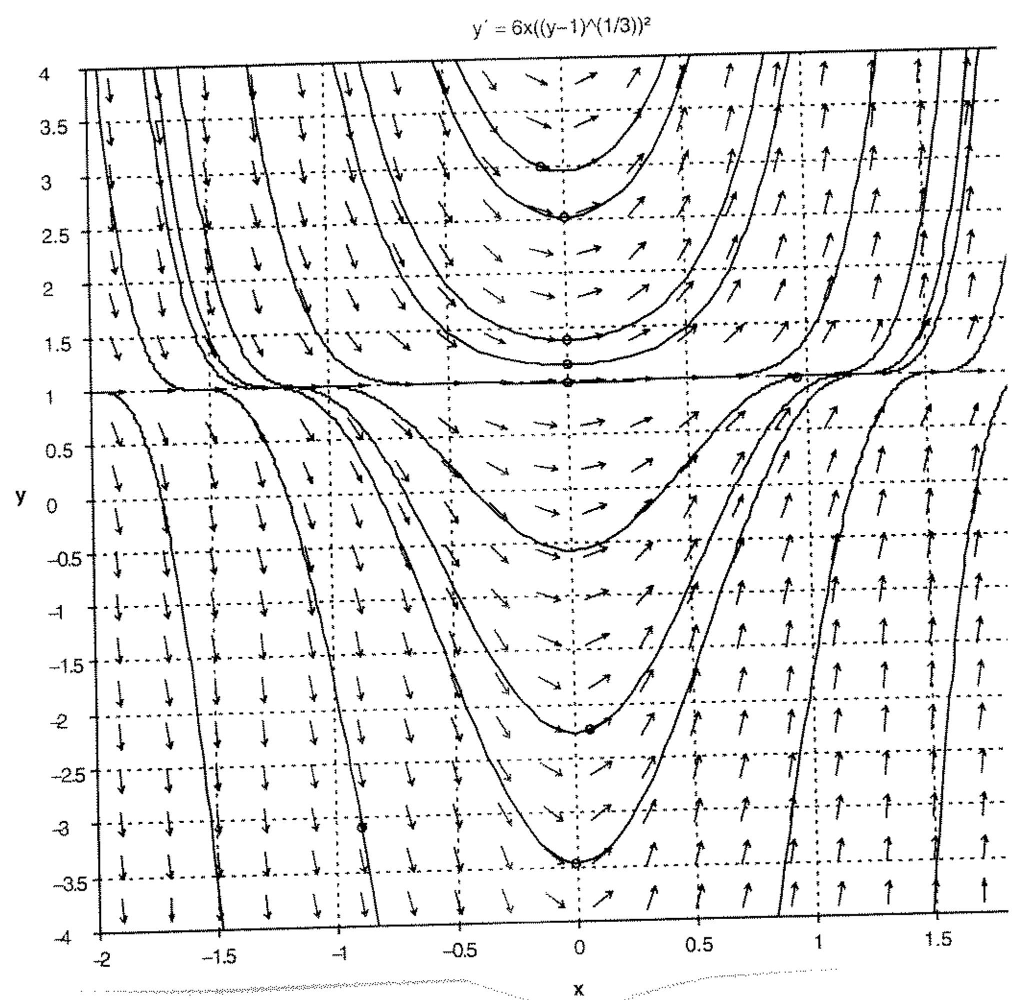
On Friday we discussed separable DE's, and how to solve them.
At the end of class we started discussing the existence-uniqueness theorem (p.23), page 4 of Friday notes.

Did we do Exercise 5, Friday notes?
Let's skip Exercise 6 (p. 5 Friday) - you can read about in text p. 24.
Instead:

Exercise 1a: (p.35) Find all solutions to $\frac{dy}{dx} = 6x(y-1)^{2/3}$

1b) Notice the singular solutions
means the sol'ns which do not appear from the separation of variables algorithm.

1c) What does this example have to do with the existence-uniqueness theorem?



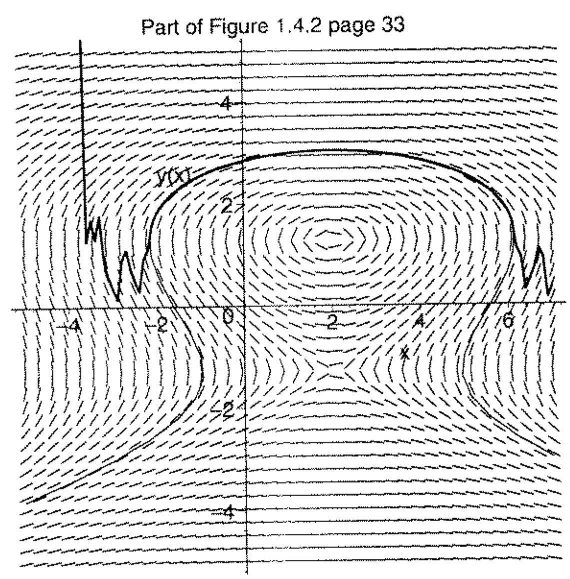
Exercise 2a) Solve the DE $\frac{dy}{dx} = \frac{4-2x}{3y^2-5}$ (p. 33)

2b) Where does existence-uniqueness theorem have issues?

2c) Discuss ways in which the "implicit" solutions could be made explicit

2d) Discuss Maple output

```
>
> with(DEtools):
> deqtn:=diff(y(x),x)=(4-2*x)/(3*y(x)^2-5):
  part1:=DEplot(deqtn,y(x),x=-5..7,y=-5..5,{[y(1)=3]},arrows=line,
     color=black,linecolor=black,dirgrid=[40,40],stepsize=.1,
     title=`Part of Figure 1.4.2 page 33 `):
  with(plots):
  part2:=implicitplot(y^3-5*y=4*x-x^2+9,x=-5..7,y=-5..5,color=black)
  :
  display({part1,part2});
```

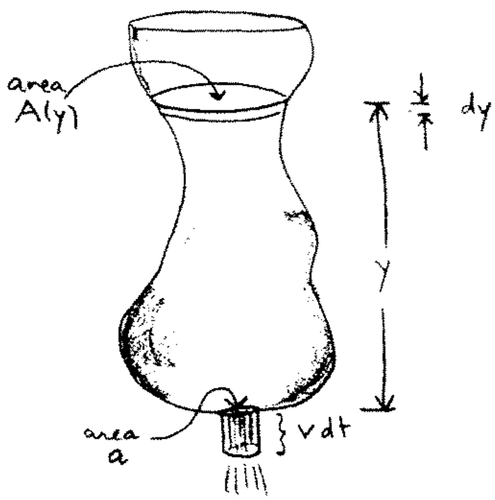


In text & hw are exponential growth/decay
+ Newton's law of cooling problems.

We did a problem like this last week ("dead body")
and the exponential growth/decay should be reminiscent of Calculus.

Let's do a separable DE application/experiment that might be new to you!

Torricelli's Law for draining tanks:



the speed v with which water leaves hole is

$$\boxed{v=\sqrt{2gy}}$$

reason: KE + PE = const
in a small time interval dt
a mass of water

$$dM = \rho\, dV = \rho\, A(y)\, dy$$

is lost from the top; replaced with equal mass

$$dM = \rho\, dV = \rho\, a\, v\, dt$$

shooting from bottom.
Since

loss in PE = gain in KE

$$\cancel{(dM)}\, g y = \tfrac{1}{2}\cancel{(dM)}\, v^2$$

$$v=\sqrt{2gy} \quad \blacksquare$$

We can express Torricelli as a separable DE by equating to two expressions for dM (or dV) on the right:

$$A(y)\, dy = a\, v\, dt$$

$$A(y)\, dy = a\sqrt{2gy}\, dt$$

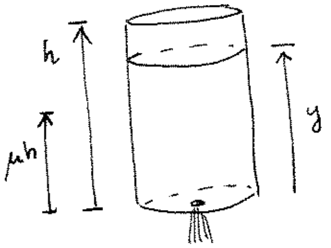
$$\boxed{A(y)\frac{dy}{dt} = -k\sqrt{y}}$$

separable DE!

Experiment!!

Cylindrical cistern: $A(y) = A$ const.

$$\frac{dy}{dt} = -k y^{1/2} \qquad \text{(different } k\text{)}.$$



Let T_μ be the amount of time it takes the cistern to empty from full height h, to height μh $(0 \le \mu \le 1)$. Show the time it takes to empty the tank is given by

$$\boxed{T_0 = \frac{T_\mu}{1-\mu^{1/2}}}$$

Nalgene bottle experiment:

I marked off the bottle so that we can use $\mu = .5$

So if we time how long it takes to empty half the height, and call it $T_{1/2}$, then the total time estimate will be

$$T_0 = \frac{1}{1-\sqrt{.5}}\, T_{.5} \approx (3.41)\, T_{.5}$$

Experiment

$T_{1/2} =$

$(3.41)\, T_{1/2} =$

$T_0 =$