Math 2250
Friday Sept 19

- Do the page 5 examples from Wed.

General set-up:

Linear system of m eqtns in n unknowns:

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}\right\} LS$$

Goal: find all (unknown) solution vectors $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ (for which all m eqtns hold simultaneously)

the collection of these solutions is called the solution set

The rectangle of coefficients

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$ is called the coefficient matrix

If we adjoin the right-hand side vector:

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & & a_{mn} & b_m \end{array}\right]$$ this is the augmented matrix

We may do the following equation operations without changing the solution set:

1. Interchange 2 eqtns
2. multiply an eqtn by a non-zero constant
3. replace an eqtn by its sum with any multiple of another eqtn

[we can combine several of these into a single step, e.g. replace an eqtn with the sum of a non-zero mult of it, and a multiple of another eqtn]

HW for Friday Sept 26

3.1 1, 4, 11, 16, 17, 19, 23, 29, 32, 33, 34
3.2 7, 8, 9 13, 17 29, 30
3.3 13, 20, 33, 34
3.4 3, 5, 7, 10, 13, 19, 31 32, 34, 39, 40 44

do these problems by hand, but I encourage software (Maple, Matlab etc.) checks.
hand in technology checks for
3.3 20 (reduced row ech. form)
3.4 13

So when we work synthetically with the augmented matrix we may do the corresponding 3 elementary row operations without changing sol'n set:

1. interchange 2 rows ("swap")
2. multiply a row by a non-zero constant
3. replace a row by its sum with a multiple of another row.

Example 5 p. 160, illustrating Gaussian elimination for a big system)

$$x_1 - 2x_2 + 3x_3 + 2x_4 + x_5 = 10$$
$$2x_1 - 4x_2 + 8x_3 + 3x_4 + 10x_5 = 7$$
$$3x_1 - 6x_2 + 10x_3 + 6x_4 + 5x_5 = 27$$

$$\begin{array}{r|rrrrr|r}
 & 1 & -2 & 3 & 2 & 1 & 10 \\
 & 2 & -4 & 8 & 3 & 10 & 7 \\
 & 3 & -6 & 10 & 6 & 5 & 27 \\
\hline
 & 1 & -2 & 3 & 2 & 1 & 10 \\
-2R_1+R_2 & 0 & 0 & 2 & -1 & 8 & -13 \\
-3R_1+R_3 & 0 & 0 & 1 & 0 & 2 & -3 \\
\hline
 & 1 & -2 & 3 & 2 & 1 & 10 \\
R_3 & 0 & 0 & 1 & 0 & 2 & -3 \\
R_2 & 0 & 0 & 2 & -1 & 8 & -13 \\
\hline
 & 1 & -2 & 3 & 2 & 1 & 10 \\
 & 0 & 0 & 1 & 0 & 2 & -3 \\
-2R_2+R_3 & 0 & 0 & 0 & -1 & 4 & -7 \\
\hline
 & 1 & -2 & 3 & 2 & 1 & 10 \\
 & 0 & 0 & 1 & 0 & 2 & -3 \\
-R_3 & 0 & 0 & 0 & 1 & -4 & 7
\end{array}$$

at this point we can backsolve to find sol'n:

$x_5 = t$
$x_4 = 7 + 4t$
$x_3 = -3 - 2t$
$x_2 = s$
$x_1 = 10 - t - 2(7+4t) - 3(-3-2t) + 2s$
$\quad = 5 + t(-3) + s(2)$

so $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \cancel{10}\,5 \\ 0 \\ -3 \\ 7 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ -2 \\ 4 \\ 1 \end{bmatrix}$, $s, t \in \mathbb{R}$

<u>row echelon form</u>

1. all "zero rows" (having all 0's) lie beneath all "non-zero rows"
2. the leading non-zero entry in each non-zero row lies strictly to the right of the leading non-zero entry in the previous row.

or proceed to reduced row-echelon form, by cleaning from the bottom right, moving up & to the left

$$\begin{array}{r|rrrrr|r}
-2R_3+R_1 & 1 & -2 & 3 & 0 & 9 & -4 \\
 & 0 & 0 & 1 & 0 & 2 & -3 \\
 & 0 & 0 & 0 & 1 & -4 & 7 \\
\hline
-3R_2+R_1 & 1 & -2 & 0 & 0 & 3 & 5 \\
 & 0 & 0 & 1 & 0 & 2 & -3 \\
 & 0 & 0 & 0 & 1 & -4 & 7
\end{array}$$

now, backsolving is easier!

$x_5 = t$
$x_4 = 7 + 4t$
$x_3 = -3 - 2t$
$x_2 = s$
$x_1 = 5 - 3t + 2s$

same ans!

<u>reduced row echelon form</u> 1, 2 <u>and</u>

3. each leading non-zero entry is a 1 (called a "leading 1")
4. Every column with a leading 1 has zeroes everywhere else, i.e. above as well as below

With Gaussian elimination we can reduce any matrix to its r.r.e.f.

Math 2250-1
reduced row echelon form
Friday 9/19/08

```
> with(linalg):  #load linear algebra package
> A:=matrix(3,6,[1,-2,3,2,1,10,2,-4,8,3,10,7,
            3,-6,10,6,5,27]); #3 rows, 6 columns
  #is the dimension, then enter successive entries
  #row by row, from left to right
```

$$A := \begin{bmatrix} 1 & -2 & 3 & 2 & 1 & 10 \\ 2 & -4 & 8 & 3 & 10 & 7 \\ 3 & -6 & 10 & 6 & 5 & 27 \end{bmatrix}$$

```
> rref(A); #get the reduced row echelon form
```

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 3 & 5 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{bmatrix}$$

You can check your hw exercises with technology, very quickly. For example, one of your exercises this week is 3.2 #17....the original system is

$$x_1 - 4\,x_2 - 3\,x_3 - 3\,x_4 = 4$$
$$2\,x_1 - 6\,x_2 - 5\,x_3 - 5\,x_4 = 5$$
$$3\,x_1 - x_2 - 4\,x_3 - 5\,x_4 = -7$$

Here is the augmented matrix and its reduced row echelon form. From this, read off the solution set!

```
> A:=matrix(3,5,[1,-4,-3,-3,4,2,-6,-5,-5,5,3,-1,-4,-5,-7]);
```

$$A := \begin{bmatrix} 1 & -4 & -3 & -3 & 4 \\ 2 & -6 & -5 & -5 & 5 \\ 3 & -1 & -4 & -5 & -7 \end{bmatrix}$$

```
> rref(A);
```

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 1 & 3 & 5 \end{bmatrix}$$

```
>
```