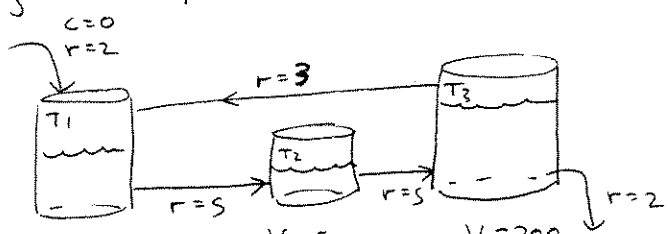
Math 2250-1
Wed Sept 17

if you want to begin the 3.1–3.2 hw (part of assignment due next Friday Sept 26:
3.1 1, 4, 11, 16, 17, 19, 23, 29, 32, 33, 34
3.2 7, 8, 9 13, 17 29, 30

Digression to linear algebra – chptrs 3-4

Why?
This is the context we need to discuss more complicated systems of DE's
(also higher order DE's)

e.g. tank systems & spring systems



vols	$V_1=100$	$V_2=50$	$V_3=200$
solute amts	$x_1(t)$	$x_2(t)$	$x_3(t)$

$$\frac{dx_1}{dt} = 3\frac{x_3}{V_3} - 5\frac{x_1}{V_1} = -\frac{5}{100}x_1 + \frac{3}{200}x_3$$

$$\frac{dx_2}{dt} = \frac{5x_1}{V_1} - 5\frac{x_2}{V_2} = \frac{5}{100}x_1 - \frac{5}{50}x_2$$

$$\frac{dx_3}{dt} = 5\frac{x_2}{V_2} - 5\frac{x_3}{V_3} = \frac{5}{50}x_2 - \frac{5}{200}x_3$$

Matrix form.

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{20} & 0 & \frac{3}{200} \\ \frac{1}{20} & -\frac{1}{10} & 0 \\ 0 & \frac{1}{10} & -\frac{1}{40} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{cases} \frac{d\vec{x}}{dt} = A\vec{x} \\ \vec{x}(t_0) = \vec{x}_0 \end{cases}$$

$$\left(\vec{x}(t) = e^{At}\vec{x}_0 \ ! \right)$$

§3.1-3.2

So, we begin by recalling systems of linear equations, and move forward from there, eventally return to DEs

1st do single eqtns

geometric sol'n set

1 linear eqtn in 1 unknown "x":	$ax=b$	
1 linear eqtn in 2 unknowns "x,y":	$ax+by=c$	
1 linear eqtn in 3 unknowns "x,y,z":	$ax+by+cz=d$	

examples

$3x = 5$

$2x + 3y = 6$

$2x + 3y + 4z = 12$

2 linear eqtns in 2 unknowns:

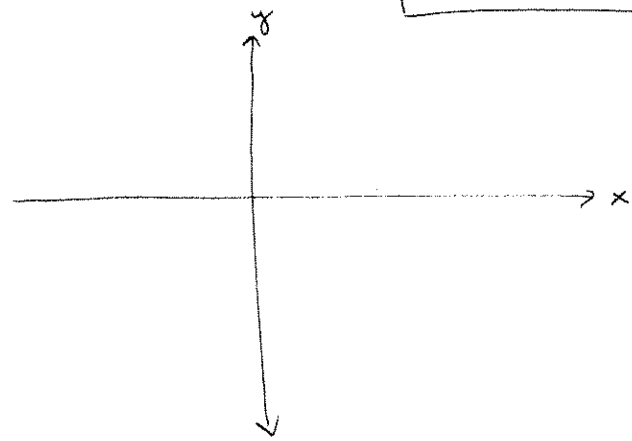
geometric solution set(s)

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

goal: find all $\begin{bmatrix} x \\ y \end{bmatrix}$ s.t. both eqtns hold

"simultaneous solution"

example



$E_1 \quad 5x + 3y = 1$

$E_2 \quad x - 2y = 8$

systematic sol'n algorithm:

$E_2 \quad x - 2y = 8$

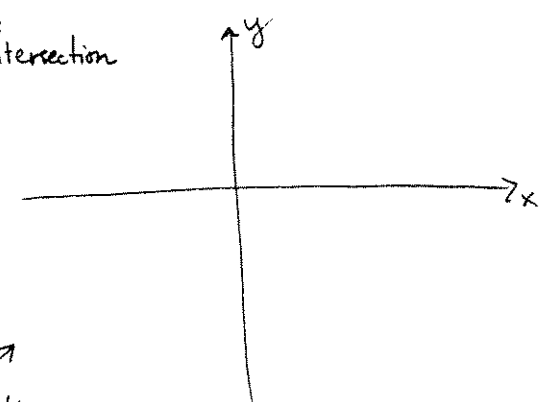
$E_1 \quad 5x + 3y = 1$

(1) Interchanging order of equations does not change solution set → same lines

$E_1 \quad x - 2y = 8$

$-5E_1 + E_2 \quad 13y = -39$

(2) replacing an eqtn by its sum with a multiple of another eqtn does not change solution set! [Why? this needs thinking!] → new lines same intersection



$E_1 \quad x - 2y = 8$

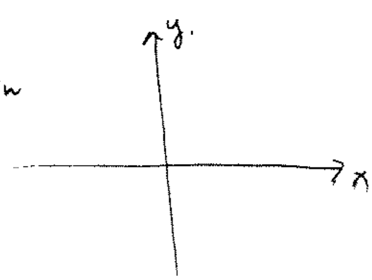
$\frac{E_2}{13} \quad y = -3$

(3) multiplying an eqtn by non-zero const does not change sol'n set → same lines

$2E_2 + E_1 \quad x = 2$

$E_2 \quad y = -3$

(2) again



We could've saved a lot of time by doing this "synthetically"

$$\left[\begin{array}{cc|c} 5 & 3 & 1 \\ 1 & -2 & 8 \end{array}\right]$$

$$\begin{array}{l} R_2 \\ R_1 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 5 & 3 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ -5R_1 + R_2 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 13 & -39 \end{array}\right]$$

$$\begin{array}{l} \\ R_2/13 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 1 & -3 \end{array}\right]$$

$$\begin{array}{l} -2R_2 + R_1 \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right]$$

$x = 2$
$y = -3$!

So, what are possible sol'n sets for 2 (linear) eqtns in 2 unknowns?

How about 3, eqtns in <u>2 unknowns</u>?
4, 5, etc

<u>example</u>: Consider the 2^{nd} order I.V.P.

$$\begin{cases} y'' - 9y = 0 \\ y(0) = 7 \\ y'(0) = 9 \end{cases}$$

Show $y(x) = Ae^{3x} + Be^{-3x}$ is always a sol'n (A, B consts)

$$y' =$$

$$y'' =$$

$$y'' - 9y =$$

Solve the IVP: (you'll need to solve a linear system of 2 equations to get A, B.)

simultaneous sol'ns to linear eqtns in 3 unknowns : geometric meaning?

$$\begin{aligned} x + 2y + z &= 4 \\ 3x + 8y + 7z &= 20 \\ 2x + 7y + 9z &= 23 \end{aligned}$$

$$\begin{array}{l} \\ -3E_1+E_2 \\ -2E_1+E_2 \end{array} \quad \begin{aligned} x + 2y + z &= 4 \\ 0 + 2y + 4z &= 8 \\ 0 + 3y + 7z &= 15 \end{aligned}$$

$$\begin{array}{l} \\ E_2/2 \\ \\ \end{array} \quad \begin{aligned} x + 2y + z &= 4 \\ y + 2z &= 4 \\ 3y + 7z &= 15 \end{aligned}$$

$$\begin{array}{l} \\ \\ -3E_2+E_3 \end{array} \quad \begin{aligned} x + 2y + z &= 4 \\ y + 2z &= 4 \\ z &= 3 \end{aligned}$$

$\leftarrow z = 3$
so $y = 4-6 = -2$
so $x = 4-3+4 = 5$

or continue $-E_3+E_1$, $-2E_3+E_2$

$$\begin{aligned} x + 2y &= 1 \\ y &= -2 \\ z &= 3 \end{aligned}$$

$$\begin{array}{l} -2E_1+E_2 \\ \\ \\ \end{array} \quad \begin{aligned} x &= 5 \\ y &= -2 \\ z &= 3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 3 & 8 & 7 & 20 \\ 2 & 7 & 9 & 23 \end{array}\right]$$

$$\begin{array}{l} \\ -3R_1+R_2 \\ -2R_1+R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 2 & 4 & 8 \\ 0 & 3 & 7 & 15 \end{array}\right]$$

$$\begin{array}{l} \\ R_2/2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 3 & 7 & 15 \end{array}\right]$$

$$\begin{array}{l} \\ \\ -3R_1+R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\begin{array}{l} -R_3+R_1 \\ -2R_3+R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\begin{array}{l} -2R_2+R_1 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

so $x = 5$, $y = -2$, $z = 3$

check ans!

geometric picture?

other possibilities! (Almost same system as page 4)

$$\begin{aligned} x + 2y + z &= 4 \\ 3x + 8y + 7z &= 20 \\ 2x + 7y + 8z &= \begin{cases} 20 \\ 23 \end{cases} \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 3 & 8 & 7 & 20 \\ 2 & 7 & 8 & \begin{cases} 20 \\ 23 \end{cases} \end{array}\right]$$

$$\begin{array}{l} \\ -3R_1+R_2 \\ -2R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 2 & 4 & 8 \\ 0 & 3 & 6 & \begin{cases} 12 \\ 15 \end{cases} \end{array}\right]$$

$$\begin{array}{l} \\ R_2/2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 3 & 6 & 12 \end{array}\right]$$

$$\begin{array}{l} \\ \\ R_3/3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & 2 & 4 \end{array}\right]$$

$$\begin{array}{l} \\ \\ -R_2+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} -2R_2+R_1 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$z = t$ (arbitrary)

$y = 4 - 2t$

$x = -4 + 3t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4+3t \\ 4-2t \\ t \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

Line of sol'ns!

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 3 & 6 & 15 \end{array}\right]$$

$$\begin{array}{l} \\ \\ -3R_2+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

?? $0 = -1$

NO SOL'N