Math 2250-1
Friday Sept. 12

HW for Fri Sept 19

- Maple project 1 is due!! (best to finish this earlier, though) — LCB 115, 1-4 p.m. Saturday (tomorrow!)
- 2.3 2,3, 9, 10,12 17,18
  2.4 5
  2.5 5,25
  2.6 5

I've posted notes on converting $A\cos\omega t + B\sin\omega t$ into $C(\cos(\omega t-\alpha))$.

Finish discussing the harvest of a logistic population, and then do §2.3 : acceleration-velocity models:

In calc: (and §1.2)

$\uparrow y$ $\quad m\frac{dv}{dt} = F_G = -mg$

$\frac{dv}{dt} = -g$

$v = -gt + v_0$

$y = -\frac{1}{2}gt^2 + v_0 t + y_0$

add resistance?

$$m\frac{dv}{dt} = F_G + F_f$$

$|F_f| \cong k|v|^p \quad 1 \le p \le 2$ empirical

$\underline{p=1}$ ("linear" model)

$$m\frac{dv}{dt} = -mg - kv$$

force will cause an acceleration opposite to direction of motion

... this is low speed "linearization"

if $F_f = F(v) = F(0) + F'(0)v + \frac{1}{2!}F''(0)v^2 + \dots$; $F(0) = 0$

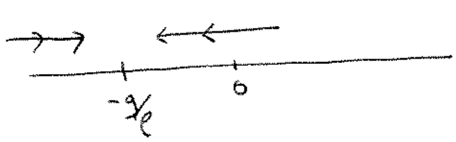
for $|v|$ small, "negligable"

$\frac{dv}{dt} = -g - \rho v \qquad \rho := \frac{k}{m}$

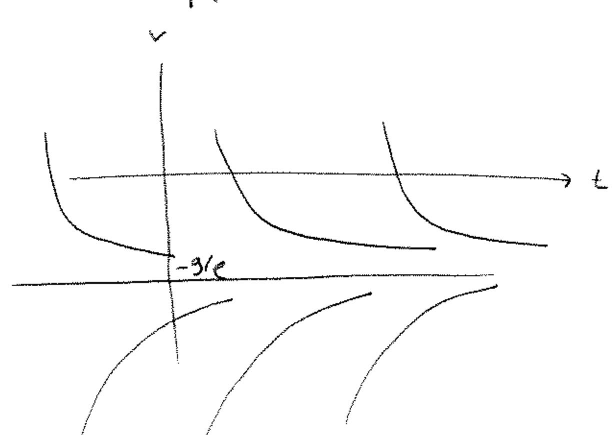
$= -\rho(v + \frac{g}{\rho})$

equil soltn: $v = -g/\rho$

phase portrait:



slope field:



analytic sol'n

$$\frac{dv}{dt} = -\rho\left(v + \frac{g}{\rho}\right)$$

$$\frac{dv}{v + g/\rho} = -\rho\, dt$$

$$\ln|v + g/\rho| = -\rho t + C \qquad t = 0 \Rightarrow C = \ln|v_0 + g/\rho|$$

$$= -\rho t + \ln|v_0 + g/\rho|$$

$$|v + g/\rho| = |v_0 + g/\rho| e^{-\rho t}$$

$$v + g/\rho = (v_0 + g/\rho) e^{-\rho t} \qquad \text{(why?)}$$

$$\boxed{v = \underbrace{-g/\rho}_{v_T \text{ "terminal velocity"}} + \underline{(v_0 + g/\rho) e^{-\rho t}}}$$

$$y = \int v(t)\,dt = t v_T + \left(\frac{v_0 - v_T}{-\rho}\right) e^{-\rho t} + C \qquad y_0 = \frac{v_T - v_0}{\rho} + C$$

so $C = y_0 + \frac{v_0 - v_T}{\rho}$

$$\boxed{y = t v_T + \left(\frac{v_0 - v_T}{\rho}\right)\left(1 - e^{-\rho t}\right) + y_0}$$

Examples 1 & 2 p. 98-100

crossbow bolt

$v_0 = 49$ m/s

$g = 9.8$ m/s^2

$y_0 = 0$

no friction

$y = -4.9 t^2 + 49 t$

$v = -9.8 t + 49$

max ht at $t = 5$ sec

$y_{max} = y(5) = 49(2.5)$

$= 122.5$ m

time aloft = 10 sec.

linear drag

$\rho = .04$ (drag coeff; empirical)

corresponds to

$|v_T| = \frac{g}{\rho} = \frac{9.8}{.04} = 245$ m/sec

(you could measure this to deduce ρ)

$v_0 - v_T = 49 + 245 = 294$

so

$v = -245 + 294 e^{-.04t}$

$v = 0$ at $\frac{245}{294} = e^{-.04t}$

$t_{max} \approx 4.56$ sec

$y = -245 t + (294)(25)(1 - e^{-.04t})$

$y(t_{max}) = ?$

When does bolt hit ground?

Computation sheet for Example 2, section 2.3

Time of maximum height:

```
> 25*ln(294.0/245);
  solve(-245+294*exp(-.04*t)=0,t); #should be same
```

$$4.558038920$$

$$4.558038920$$

Formulas for height and velocity functions

```
> v:= t -> 294*exp(-.04*t) - 245;
```

$$v := t \to 294\,\mathrm{e}^{(-0.04\,t)} - 245$$

```
> y:= t-> -245*t + 294*25*(1 - exp(-.04*t));
```

$$y := t \to -245\,t + 7350 - 7350\,\mathrm{e}^{(-0.04\,t)}$$

Maximum height, return time tor ground time spent falling, landing speed:

```
> y(4.558038920); #max height
```

$$108.280465$$

```
> solve(y(t)=0,t);  #find when returns to ground
```

$$9.410949931,\ 0.$$

```
> 9.410949931 - 4.558038920; #time descending
```

$$4.852911011$$

```
> v(9.410949931); #speed when it lands
```

$$-43.2273093$$

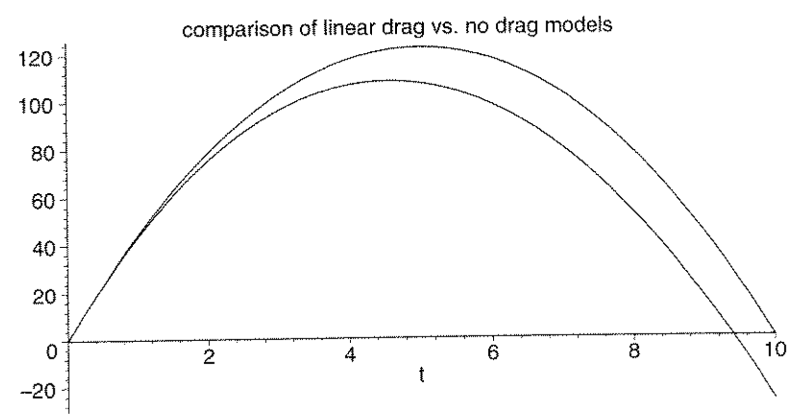
Conclusions: bolt rises for 4.56 seconds, to a height of 108.3 meters. Then it spends 4.85 seconds descending, landing with a velocity of -43.3 meters per second.

```
> with(plots):
Warning, the name changecoords has been redefined
```

```
> z:= t->-4.9*t^2 + 49*t;  #the no resistance model
```

$$z := t \to -4.9\,t^2 + 49\,t$$

```
> plot({z(t),y(t)}, t = 0..10, color=black,
  title=`comparison of linear drag vs. no drag models`);
```



quadratic drag is also interesting...

going up:

$$m\frac{dv}{dt} = -mg - kv^2$$

$$\frac{dv}{dt} = -g\left(1+\frac{k}{g}v^2\right)$$

$$\frac{dv}{1+\frac{k}{g}v^2} = -g\,dt \;..$$

arctan!

going down:

$$m\frac{dv}{dt} = -mg + kv^2$$

$$\frac{dv}{dt} = -g\left(1-\frac{k}{g}v^2\right)$$

$$\frac{dv}{1-\frac{k}{g}v^2} = -g\,dt$$

parfrac! (or $\tanh^{-1}$)