Math 2250-1
Wednesday Sept 10

* autonomous DE's,
  equilibrium sol'ns,
  stability.

examples and discussion page 3 Tuesday notes

additional examples:

Find equilibria and discuss stability for

$$\frac{dx}{dt} = x^4 - x^2$$

$$\frac{dx}{dt} = \cos x$$

* page 4 Tuesday notes: explosion-extinction models, and concrete (numerical) example.

* I will be in the Math Dept computer classroom LCB 115 from 1–4 p.m. Saturday, so that students may work on their Maple projects.

this is not the one in student center, it's down the hall in LCB.

Warning: our terminals do NOT accept memory devices, so if you have a Maple file email it to an account you can access with an internet browser.

Warning: your Math Dept login name/psswrd are not your uid/psswrd. I know how to construct your login name & default password, but won't be able to help you if you've created a new password & forgotten it.

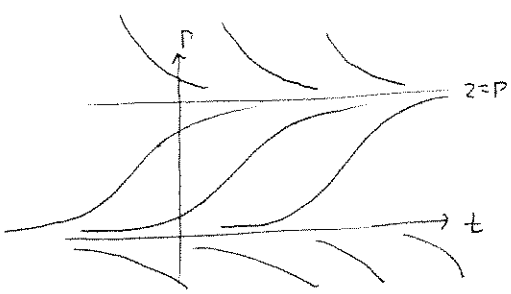
Now, let's harvest a logistic population! (e.g. fisheries).

$$\frac{dP}{dt} = \underbrace{aP - bP^2}_{\text{logistic}} - \underbrace{h}_{\text{constant rate harvesting}}$$

constant rate harvesting. (see §2.2 #24)

one could also consider a term $-hP$, which would maybe correspond to constant effort harvesting (see §2.2 #23)

e.g. $\frac{dP}{dt} = 2P - P^2 = P(2-P)$



vs $\frac{dP}{dt} = 2P - P^2 - h$

consider what happens for different h values

roots of RHS: $P^2 - 2P + h = 0$ has roots

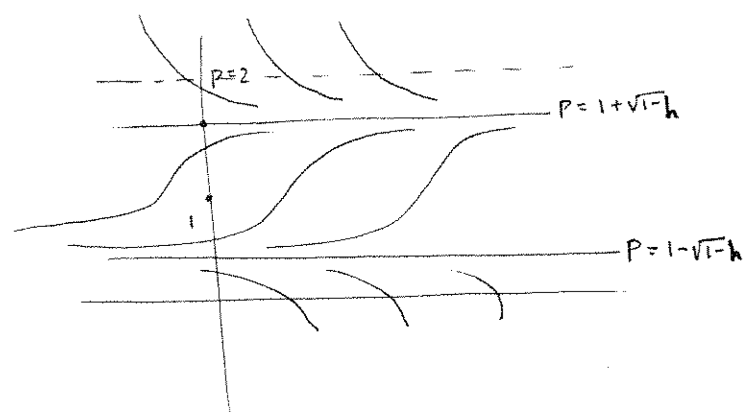
$$P = \frac{2 \pm \sqrt{4-4h}}{2} = 1 \pm \sqrt{1-h}$$

so, for $0 \le h \le 1$,

$$\frac{dP}{dt} = -\left(P - (1+\sqrt{1-h})\right)\left(P - (1-\sqrt{1-h})\right)$$

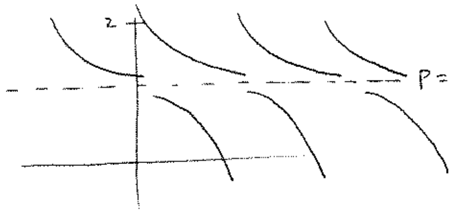
still looks logistic (& fishery won't collapse) if $h \ll 1$

extinction zone

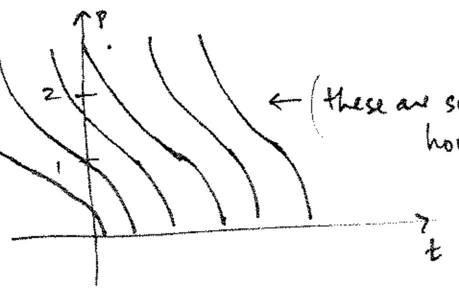


as $h \nearrow 1^-$ the middle zone (where populations increase to the modified carrying capacity) shrinks in width, until at $h = 1$ the two roots coalesce;

$$\frac{dP}{dt} = 2P - P^2 - 1 = -(P^2 - 2P + 1) = -(P-1)^2$$



and for $h > 1$, $2P - P^2 - h = -(P^2 - 2P + h) = -((P-1)^2 + h-1) < 0 \quad \forall P$ (but least negative) @ $P=1$

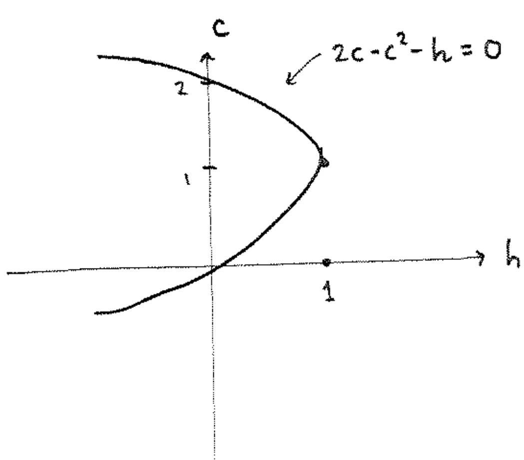


← (these are supposedly horizontal translations of each other)

Extinction $\forall P_0$!!

this model gives a plausible explanation for why fishery after fishery has collapsed around the world - if $h < 1$ but near 1, and if something perturbs the system a little bit (e.g. increased fishing pressure, a big storm, etc.), you could be confronted with "sudden", unexpected fishery collapse.

"bifurcation diagram" of equilibrium sol'ns, in the h-c plane:



$h < 0$ stocking!

$h > 0$ harvesting