

Review Sheet

Math 2250-1 November 14, 2008

Our exam covers chapters 4, 5, EP3.7, and 10 of the text. Only scientific calculators will be allowed on the exam. There is a practice session on this Saturday Nov. 15, in JWB 335, from 11:00-12:30. We will go over a practice exam, which will be posted by Friday at 1:00 p.m.

Chapter 4:

At most 30% of the exam will deal directly with this material....but much of Chapter 5 uses these concepts, so much more than 30% of the exam will be related to chapter 4. (And, as far as matrix computations go, you should remember everything you learned in Chapter 3.)

Know Definitions:

(a) **Vector Space:** A collection of objects which can be added and scalar multiplied, so that the usual arithmetic properties (Page 236) hold. You do not need to memorize all eight of these properties. The key point is that not only is R^n a vector space, but also certain subsets of it are, and so are spaces made out of functions...because functions can be added and scalar multiplied (page 274.)

(b) **Subspace:** a subset of a vector space which is itself of vector space....to check whether a subset is actually a subspace you only have to show that sums and scalar multiples of subset elements are also in the subset (Theorem 1 page 237.) Examples of important subspaces are the solution space to a homogeneous matrix equation (page 239), the span of a collection of vectors (page 243), AND the set of homogenous solutions to a linear differential equation (section 5.2).

(c) A **linear combination** of a set of vectors $\{v_1, v_2, \dots, v_n\}$ is any expression $c_1*v_1 + c_2*v_2 + \dots + c_n*v_n$. (page 242)

(d) The **span** of a set of vectors $\{v_1, v_2, \dots, v_n\}$ is the collection of all linear combinations. (page 243)

(e) A collection $\{v_1, v_2, \dots, v_n\}$ is **linearly dependent** if and only if some linear combination (with not all c_i 's = 0) adds up to the zero vector.

(f) A collection $\{v_1, \dots, v_n\}$ is **linearly independent** if and only if the only linear combination of them which adds up to zero is the one in which all coefficients $c_i=0$. (page 244)

(g) A **basis** for a vector space (or subspace) is a set of vectors $\{v_1, \dots, v_k\}$ which span the space and which are linearly independent. (page 249.)

(h) The **dimension** of a vector space is the number of elements in any basis.

Know Facts:

(a) If the dimension of a vector space is n , then no collection of fewer than n vectors can span and every collection with more than n elements is dependent.

(b) n vectors in R^n are a basis if and only if the square matrix in which they are the columns is non-singular. So you can use \det or $rref$ as a test for basis in this case.

(c) Basically all linear independence and span questions in R^n can be answered using $rref$. (see below.)

(d) You can toss dependent vectors out of a collection without changing the span. In this manner you can take a spanning set and turn it into a basis.

Do computations:

(a) Be able to check whether vectors are independent or dependent, e.g. problems page 248. (4.3) Know how to use $rref$ to check for dependencies.

(b) Be able to find bases for the solution space to homogeneous equations, e.g. problems page 255 (4.4)

Chapter 5 and EP3.7.

At least 50% of the exam will cover this material, and at least 30% of the exam will be from sections 5.4 and 5.5. (Answering questions from 5.4 and 5.5 almost always uses 5.1-5.3 material implicitly.)

5.1-5.3, 5.5 General theory:

Linear differential equations (page 296.)

principle of superposition (e.g. Theorem 1 page 296, also leads to the fact that the general solution y to the inhomogeneous equation is $y_p + y_h$, where y_p is a particular solution, and y_h is the general solution to the homogeneous equation. (Theorem 5 page 306.) Also leads to a method for getting particular solutions which are sums of particular solutions for pieces of the right hand side.)

homogeneous ($L(y)=0$). Solution space is an n -dimensional vector space. Know how to find it for constant coefficients, using exponentials and the resulting **characteristic equation** and **Euler formula** if necessary (section 5.3 and problems). What to do with repeated roots. The Wronskian test for linear independence .

nonhomogeneous ($L(y)=f$). Know how to find particular solutions by the method of undetermined coefficients. (Section 5.5 and problems) (But variation of parameters will not be on the exam.)

initial value problem, existence and uniqueness. Know how to solve initial value problems by finding y_p , and y_h , and then finding values of constants in y_h to match initial conditions.

5.4, 5.6, EP3.7 Mechanical vibrations and forced oscillations; electrical circuit analog

unforced oscillations (i.e. solutions to the homogeneous DE):

undamped (simple harmonic motion)

going from $A*\cos(\omega t) + B*\sin(\omega t)$ to $C*\cos(\omega t - \alpha)$. (The ABC triangle, amplitude and phase.)
derivation of spring equation from Newton's and Hooke's Laws.

damped.

under-damped, over-damped, critically damped. Know how to recognize, and different forms of the solution.

forced oscillations:

undamped:

resonance, and when it arises. form of solution, as follows from general theory above.

beating, when ω is close to ω_0 .

damped:

general solution is sum of **steady state periodic**, with **transient**. How to find each piece, and express the

steady state periodic solution in amplitude- phase form.

practical resonance, will occur if damping is small and driving frequency is near natural frequency.

An **RLC** circuit corresponds to a mass-spring configuration, where L corresponds to mass, R to damping coefficient, and $1/C$ to spring constant.

Chapter 10:

At least 30% of the exam will deal directly with this material. The most efficient way for me to test it is to have you solve IVPs from chapter 5, using chapter 5 techniques as well as Laplace transform techniques. You will be responsible for 10.1-10.4 on the exam (not 10.5 which we surveyed quickly and in which you worked several homework problems). You will be provided with the Laplace transform table on the front book cover. The one thing that table is missing is the definition of Laplace transform, which you should memorize (page 568).

Key skills:

Using the definition of Laplace transform to reproduce a table entry or to compute the transform of a function not on the table.

Converting an initial value problem into an algebraic equation for the Laplace transform of the solution (section 10.2).

Using the table to convert in both directions between functions and their Laplace transforms. (covers 10.1-10.5 facts)

Setting up and using partial fractions (and possibly **completing the square**) to simplify Laplace transforms so that you can use the table to invert them. (section 10.3)

Using and computing convolutions to invert the product of two Laplace transforms (section 10.4).

Using the **unit step function** to turn forcing functions on and off in spring configuration DEs (section 10.5) is important, but won't be on the exam.