

Math 2250-01
Mechanical Oscillations - Maple Project 2
Notes and assignment
Monday October 20

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Project Directions: This project deals with concepts discussed in Chapter 5. It would be best to work through this project with your book in hand. Ms. Taylor (2250-3) adapted this project from Professor Gustafson's version (2250-2), and I made a few minor changes to her adaptation. There are three parts to the project, and each part has a few questions associated with it. Hand in a single Maple document containing your answers. Clearly label each of your answers if you want to receive the credit for them. (A good way to create your solution file would be to start with this maple worksheet, and make appropriate insertions and deletions.) Some parts may call for handwritten work - make your answers legible, either by leaving yourself some space in the document to (neatly) fill in by hand, or create the answers as text displays.
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Part 1 - Underdamped Oscillations

Consider the equation for a free damped linear oscillator

$$\left[\begin{array}{l} > m \cdot \text{diff}(x(t), t, t) + c \cdot \text{diff}(x(t), t) + k \cdot x(t) = 0; \\ \\ m \left(\frac{d^2}{dt^2} x(t) \right) + c \left(\frac{d}{dt} x(t) \right) + k x(t) = 0 \end{array} \right.$$

[with initial conditions:

$$\left[\begin{array}{l} > x(0) = 0, \quad D(x)(0) = 1; \\ \\ x(0) = 0, \quad D(x)(0) = 1 \end{array} \right.$$

where, m , c and k are non-negative constants.

Exercise 1. Using the information given to you on page 327, if $m = 3$ and $c = 4$ find (by hand) a value for k that would provide the scenario of underdamping. Explain how you arrive at this conclusion. Next, solve the characteristic polynomial (by hand or in Maple). How does this value of k affect the roots of this polynomial, and thus the dynamics of the differential equation? Now, using Maple, find and display the exact formula for the solution $x(t)$ to this initial value problem.

Exercise 2. Plot the solution $x(t)$ vs. time. (the graph should look something like the one in Figure 5.4.9 on page 328.) Then create a display which includes the solution plot as well as the decaying exponential (and its opposite), which enclose the solution graph as in Figure 5.4.9.

Exercise 3. From your plot found in Exercise 2, estimate the value of the pseudoperiod. You can click with the mouse on the graphic to print the cursor location in the left upper corner of the maple window. The coordinates printed are of the form (x,y) . From this coordinate information, you can estimate the period by simple subtraction. Next, find the exact pseudoperiod as given on page 328 of your book. Do these two values agree?

A little bit of sample code to help you on your way through Part 1 (as found on Prof. Gustafson's page...)

```
[ > restart: #clear all old definitions from memory
[ > with(DEtools): #load the DE library of commands
[ with(plots): #let's you use "plot", "display"
[ > #Note: in your code, use semicolons. And use numbers rather
[ #than letters, for the mass, spring, and
[ #damping constants.
[ > #Defining the differential equation
[ de:= m*diff(x(t), t,t) + c*diff(x(t),t) + k*x = 0:
[ > #solving the characteristic polynomial can be done with
[ solve(a*r^2+b*r+c=0,r):
[ > #Defining initial conditions
[ ic:= x(0)=d, D(x)(0)=f:
[ > #Symbolically solve for x(t)
[ p:=dsolve({de,ic}, x(t), method=laplace):
[ > #Capture the dsolve symbolic solution as a function of X(t)
[ X:= unapply(rhs(p),t):
[ > #Plot the solution - you decide what t domain is appropriate
[ plot(X(t), t=0..5):
```

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Part 2 - Undamped Forced Oscillations

Consider the undamped (c=0) forced problem

```
[ > m*diff(x(t), t,t) + k*x(t) = 5*cos(omega*t);
```

$$m \left(\frac{d^2}{dt^2} x(t) \right) + k x(t) = 5 \cos(\omega t)$$

```
[ with initial conditions
[ > x(0)=0, D(x)(0)=0;
```

$$x(0)=0, D(x)(0)=0$$

where m , k and w are non-negative constants. For this problem, assume that $m=3$ and $k=4.5$. Start by reading through pages 349-352 so you know where you are headed.

Exercise 4. Choose the forcing angular frequency to be 3 times larger than the natural angular frequency (see page 349) . Solve for $x(t)$ using `dsolve`. Plot the solution on a suitable interval in order to show the global behavior of the solution $x(t)$. You should get something that looks like Figure 5.6.2 on page 350.

Exercise 5. The solution $x(t)$ is the sum of two functions, one of period $2\pi/w$ and the other of period $2\pi/w_0$. Using your solution to $x(t)$ from Exercise 4 (and page 350 as a guide), what is the value of the exact period?

Exercise 6. Suggest a value for the forcing frequency so that the oscillations exhibit resonance (show how you arrive at this value). Make a graph of resonant behavior as in Figure 5.6.4 on page 352.

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Tips for Part 2- With a little modification, you can use the same code as in part 1.

Part 3 - Resonance

Finally, we will consider a scenario with elements from Part 1 and Part 2. Consider a damped forced problem,

$$m \frac{d^2}{dt^2} x(t) + c \frac{d}{dt} x(t) + k x(t) = 5 \cos(w t)$$

with initial conditions:

$$x(0) = 0, D(x)(0) = 0$$

and now assume that $m = 3$ and $k = 45$.

Exercise 7. Consider the damping constants $c=2$, $c=1$, and $c=1/2$. Compute the amplitude function $C(w)$ as found on page 357 of your text for these three equations, then plot three amplitude graphs on a single set of axes for $w = 0$ to 20. You should end up with three curves that look like that of Figure 5.6.9 on page 357.

Exercise 8. Using the mouse on your plot of this graph, find the values of w^* and C^* for the three separate cases, where C^* is the maximum value of the amplitude function in your plot (you should end up with a table of six values). You will find that as $c \rightarrow 0$, C^* gets larger and larger. What does this tell you about the model?

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Some sample code for Part 3

```
[ > #make sure w and c are variables, define numerical
#values for c1,c2,c3,k,m and F0, and then try:
[ > #equation (21) page 355, for steady state amplitude:
C:=(w,c)->F0/sqrt((k-m*w*w)^2+(c*w)^2):
[ > plot({C(w,c1), C(w,c2), C(w,c3)},w=0..20,color=black):
.....
```