Name: SOLUTIONS!
I.D. number

Math 2250-1
Practice Final Exam
December 2008
(Mostly the actual final exam from 2004)

This exam is closed-book and closed-note. You may use a scientific calculator, but not one which is capable of graphing or of solving differential or linear algebra equations. Integral tables and Laplace Transform Tables are included with this exam. **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** This exam counts for 30% of your course grade. It has been written so that there are 200 points possible, however, and the point values for each problem are indicated in the right-hand margin. **Good Luck!**

1) Consider the differential equation

$$P'(t) = -P^2 + 7P - 10$$

1a) Find the equilibrium solutions of this differential equation.

(2 points)

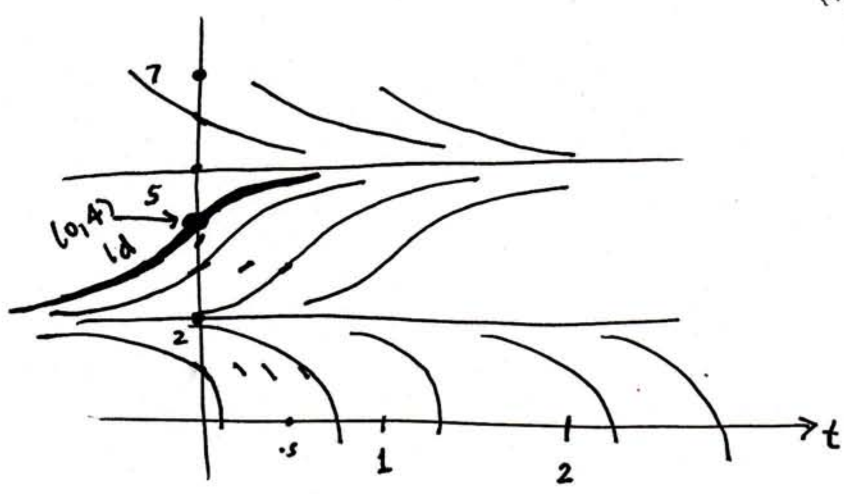
$$P'(t) = -(P^2 - 7P + 10) = -(P-5)(P-2) \qquad \boxed{P = 2, 5}$$

1b) Sketch the slope field (or enough solution trajectories that the slope field can be deduced), for this differential equation, say with 0<P<7. Include the graphs of the equilibrium solutions.

(4 points)

(phase portrait quicker!)

P	P'
0	-10
2	0
5	0
1	-4
3	2
4	2
6	-2



1c) Classify the equilibrium solutions as being stable or unstable.

(4 points)

$P = 2$ is unstable

$P = 5$ is stable (asymptotically)

1d) Use separation of variables to solve the initial value problem

$$P'(t) = -P^2 + 7P - 10$$
$$P(0) = 4$$

for P(t).

(15 points)

$$\frac{dP}{dt} = -(P-5)(P-2)$$

$$\frac{dP}{(P-5)(P-2)} = -dt$$

$$\frac{dP}{3}\left[\left(\frac{1}{P-5}\right) - \frac{1}{P-2}\right] = -dt$$

$$\frac{1}{P-5} - \frac{1}{P-2}\, dP = -3\, dt$$

$\int$: $\ln\left|\frac{P-5}{P-2}\right| = -3t + C_1$

$$\left|\frac{P-5}{P-2}\right| = e^{C_1} e^{-3t}$$

$$\frac{P-5}{P-2} = Ce^{-3t}$$

@ t=0: $\frac{4-5}{4-2} = C = -\frac{1}{2}$

$$\frac{P-5}{P-2} = -\frac{1}{2}e^{-3t}$$

$$P-5 = (P-2)\left(-\frac{1}{2}e^{-3t}\right)$$

$$P\left(1+\frac{1}{2}e^{-3t}\right) = 5 + e^{-3t}$$

$$\boxed{P(t) = \frac{5+e^{-3t}}{1+\frac{1}{2}e^{-3t}}}$$

check : $P(t) \to 5$ as $t \to \infty$

$P(0) = \frac{6}{3/2} = 4$ ✓

1e) Add (or highlight) the graph of your solution to (1c) to the slope field picture in (1b). Verify below that your formula for P(t) gives you the limit predicted by the slope field, as P approaches infinity.

(5 points)

$$\lim_{t\to\infty} \frac{5+e^{-3t}}{1+\frac{1}{2}e^{-3t}} = \frac{5+0}{1+0} = 5,$$ consistent with picture on page 1.

2a) A small motorboat with passenger has total mass of 150 kilograms, and its motor is able to provide 50 Newtons of thrust. However, when the boat is in motion, drag from the water produces a force of 5 newtons for each meter/sec of boat velocity. Use Newton's law to explain why (while the motor is on) the boat velocity satisfies the differential equation

$$v'(t) = \frac{1}{3} - \frac{1}{30}\frac{v}{}$$

(5 points)

$mv' =$ net forces

$150v' = 50 + F_{drag}$

$F_{drag} = -5v$

$150v' = 50 - 5v$ so $v' = \frac{1}{3} - \frac{5}{150}v' = \frac{1}{3} - \frac{1}{30}v$ ✓

2b) What is the equilibrium solution for the velocity? Is this solution stable or unstable? Explain the meaning of this solution in terms of the boat's motion.

(5 points)

$\frac{1}{3} - \frac{1}{30}v = 0$

$\frac{1}{3} = \frac{1}{30}v$

$\boxed{v = 10 \text{ m/s}}$

phase portrait

v

dv/dt

$-$

10

$+$

so $v=10$ is stable; it's the terminal velocity!

2c) Solve the INITIAL VALUE PROBLEM for the boat's velocity, assuming the boat starts at rest. You are to solve it TWO of the following FOUR different ways, your choice. Your choices are
(I) FIRST ORDER LINEAR (as in Chapter 1)
(II) SEPARABLE
(III) NTH ORDER LINEAR (with N=1) (as in Chapter 5, with particular and homogeneous parts to the solution)
(IV) LAPLACE TRANSFORM.

(20 points)

If you make multiple attempts you must indicate clearly which two are to count for grading.

I) $\begin{cases} v' + \frac{1}{30}v = \frac{1}{3} \\ v(0) = 0 \end{cases}$

$e^{\frac{1}{30}t}\left(v' + \frac{1}{30}v\right) = e^{\frac{1}{30}t}\frac{1}{3}$

$\left(e^{\frac{1}{30}t}v\right)' = \frac{1}{3}e^{\frac{1}{30}t}$

$e^{\frac{1}{30}t}v = \int \frac{1}{3}e^{\frac{1}{30}t}\,dt = 10e^{\frac{1}{30}t} + C$

$v = 10 + Ce^{-\frac{1}{30}t}$

$v(0) = 0 = 10 + C \Rightarrow C = -10$

$\boxed{v = 10 - 10e^{-\frac{1}{30}t}}$

II) $\frac{dv}{dt} = -\frac{1}{30}(v-10)$

$\frac{dv}{v-10} = -\frac{1}{30}dt$

$\ln|v-10| = -\frac{1}{30}t + C_1$

$|v-10| = e^{C_1}e^{-\frac{1}{30}t}$

$v - 10 = Ce^{-\frac{1}{30}t}$

$v = 10 + Ce^{-\frac{1}{30}t}$

$v(0) = 0 \Rightarrow C = -10$

$\boxed{v = 10 - 10e^{-\frac{1}{30}t}}$

III $v' + \frac{1}{30}v = \frac{1}{3}$

$v = v_p + v_H$

try $v_p = C$

$v_p' = 0$

$v_p' + \frac{1}{30}v = 0 + \frac{C}{30} = \frac{1}{3}$

$\Rightarrow C = 10$

$p(r) = r + \frac{1}{30} = 0$

$r = -\frac{1}{30}$

$v_H = Ce^{-1/30\,t}$

So $v = 10 + Ce^{-\frac{1}{30}t}$.

$v(0) = 0 \Rightarrow C = -10$

$\boxed{v = 10 - 10e^{-\frac{1}{30}t}}$

(IV) $sV(s) - \overset{0}{v_0} + \frac{1}{30}V(s) = \frac{1}{3}\frac{1}{s}$

$V(s)\left(s + \frac{1}{30}\right) = \frac{1}{3}\frac{1}{s}$

$V(s) = \frac{1}{3}\left(\frac{1}{s(s+\frac{1}{30})}\right)$

$= \frac{1}{3}30\left(\frac{1}{s} - \frac{1}{s+1/30}\right)$

$V(s) = \frac{10}{s} - \frac{10}{s+1/30}$

$\downarrow \mathcal{L}^{-1}$

$\boxed{v(t) = 10 - 10e^{-\frac{1}{30}t}}$

2d) How long does it take the boat to accelerate from rest to half of its terminal velocity?

(5 points)

solve $v(t) = \frac{1}{2}10 = 5$

$10 - 10e^{-\frac{1}{30}t} = 5$

$5 = 10e^{-\frac{1}{30}t}$

$.5 = e^{-\frac{1}{30}t}$

$\ln(.5) = -\frac{1}{30}t$

$t = -30\ln(.5)$

$= 30\ln 2$

$(\approx 20.8 \text{ sec.})$

3) Consider this initial value problem, which could arise as a spring problem with forced oscillations:

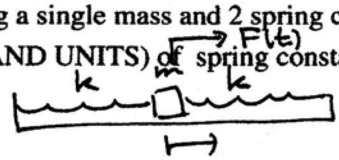
$$x''(t) + 4x = 3\cos(2t)$$

$$x(0) = 1$$

$$x'(0) = 2$$

3a) If this *was* modeling a single mass and 2 spring configuration as shown below, and if the mass m was 1 kg, what value (AND UNITS) of spring constant k would lead to the differential equation above?

(4 points)



$$mx'' = -kx - kx + F(t) \quad x(t)$$
$$= -2kx + F(t)$$

$m=1$: $\quad x'' + 2kx = F$

so $\boxed{k = 2 \ N/m}$

3b) Solve the initial value problem above, USING LAPLACE TRANSFORMS. (Hint: use the table you have been provided with to do the inverse Laplace transformations.)

(16 points)

$$x'' + 4x = 3\cos 2t$$

$$s^2 X(s) - s - 2 + 4X(s) = 3\frac{s}{s^2+4}$$

$$X(s)(s^2+4) = 3\frac{s}{s^2+4} + s + 2$$

$$X(s) = \frac{3s}{(s^2+4)^2} + \frac{s}{s^2+4} + \frac{2}{s^2+4}$$

$$\boxed{x(t) = 3\frac{t}{4}\sin 2t + \cos 2t + \sin 2t}$$

table

$k=2$

3c) What happens to the solution to this problem as t increases? What physical phenomenon is exhibited in this forced oscillation problem?

(5 points)

resonance! We forced at natural frequency, and (the first term) has amplitude growing linearly in time

4) Consider the homogeneous linear differential equation for a function x(t):

$$x'''(t) + 2\,x''(t) + 5\,x'(t) = 0$$

4a) What is the dimension of the solution space to this differential equation?

(5 points)

3-d

4b) Find a basis for the solutions to this DE.

(10 points)

$p(r) = r^3 + 2r^2 + 5r = 0$

$= r(r^2 + 2r + 5)$

$r = 0$

$x_1(t) = 1$

$(r+1)^2 + 4 = 0$

$(r+1)^2 = -4$

$r + 1 = \pm 2i$

$r = -1 \pm 2i$

or $r = \dfrac{-2 \pm \sqrt{4-20}}{2}$

$= -1 \pm 2i$

$e^{(-1+2i)t} = e^{-t}(\cos 2t + i \sin 2t)$

so $\{1, e^{-2t}\cos 2t, e^{-2t}\sin 2t\}$ is a basis.

4c) Find the general solution to the differential equation

$$x'''(t) + 2\,x''(t) + 5\,x'(t) = 10$$

(10 points)

$x_H = c_1 + c_2 e^{-2t}\cos 2t + c_3 e^{-2t}\sin 2t$

$x_p = Ct$

because C solves x_H.

$x_p' = C$

$x_p'' = x_p''' = 0$

so $Lx_p = 0 + 0 + 5C = 10$

$C = 2$

$x = x_p + x_H$

$$\boxed{x = 2 + c_1 + c_2 e^{-2t}\cos 2t + c_3 e^{-2t}\sin 2t}$$

5) Consider the matrix A defined by

$$A := \begin{bmatrix} -1 & 0 & 2 \\ 1 & -6 & 0 \\ 0 & 6 & -2 \end{bmatrix}$$

5a) Find the characteristic polynomial and factor it to find the eigenvalues of A. (Hint, the eigenvalues you get should be = 0, -4, -5; your job is to derive these values.)

(5 points)

$$|A-\lambda I| = \begin{vmatrix} -1-\lambda & 0 & 2 \\ 1 & -6-\lambda & 0 \\ 0 & 6 & -2-\lambda \end{vmatrix} \overset{\text{across top row}}{=} (-1-\lambda)\begin{vmatrix} -6-\lambda & 0 \\ 6 & -2-\lambda \end{vmatrix} - 0 + 2\begin{vmatrix} 1 & -6-\lambda \\ 0 & 6 \end{vmatrix}$$

$$= (-1-\lambda)(-6-\lambda)(-2-\lambda) + 12$$

$$= -(\lambda+1)(\lambda+6)(\lambda+2) + 12$$

$$= -(\lambda^2+7\lambda+6)(\lambda+2) + 12 = -\lambda^3 - 9\lambda^2 - 20\lambda - 12 + 12$$

$$= -(\lambda^3 + 9\lambda^2 + 20\lambda)$$

$$= -\lambda(\lambda^2 + 9\lambda + 20)$$

$$= -\lambda(\lambda+4)(\lambda+5)$$

roots $\lambda = 0, -4, -5$ ✓

5b) An eigenvector for $\lambda = 0$ is given by

$$u := \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix}$$

Find eigenvectors for the other two eigenvalues, to get a basis for R^3 made out of eigenvectors of A.

(20 points)

$\lambda = -4$

$$\left[\begin{array}{ccc|c} 3 & 0 & 2 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 6 & 2 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2 \\ -3R_2+R_1 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 6 & 2 & 0 \\ 0 & 6 & 2 & 0 \end{array}\right]$$

$$-R_2+R_3 \quad \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_3 = t$
$v_2 = -t/3$
$v_1 = -2/3\, t$

$$\vec{v} = t\begin{bmatrix} -2/3 \\ -1/3 \\ 1 \end{bmatrix} \nearrow \vec{v} = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$$

$\lambda = -5$

$$\left[\begin{array}{ccc|c} 4 & 0 & 2 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 6 & 3 & 0 \end{array}\right]$$

$$\begin{array}{l} R_2 \\ -4R_2+R_1 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 6 & 3 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_2+R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

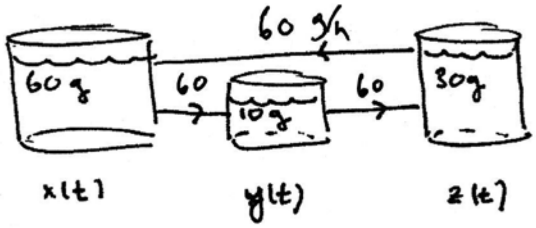
$v_3 = t$
$v_2 = -\tfrac{1}{2}t$
$v_1 = -\tfrac{1}{2}t$

$$\vec{v} = t\begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$$

6) Consider the following three-tank configuration. In tank one there is uniformly mixed volume of 60 gallons, and x(t) pounds of salt solute. In tank two there is mixed volume of 10 gallons and y(t) pounds of salt. In tank three there is 30 gallons of liquid and z(t) pounds of salt. Water us pumped slowly from tank one to tank two, from tank two to tank three, and from tank three back to tank one, and all rates are 60 gallons per hour.



6a) Model this tank configuration, to arrive at the first order system of differential equations

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -6 & 0 \\ 0 & 6 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(5 points)

$x_a' = r_i c_i - r_o c_o = 60\frac{z}{30} - 60\frac{x}{60}$
$= 2z - x$

$y' = 60\frac{x}{60} - 60\frac{y}{10}$
$= x - 6y$

$z' = 60\frac{y}{10} - 60\frac{z}{30}$
$= 6y - 2z$

so $x' = -x \quad + 2z$
$y' = x - 6y$
$z' = \quad 6y - 2z$

matrix form.

6b) Assume that at time t=0 there are 40 pounds of salt in tank 1, 20 pounds in tank 2 and 40 pounds in tank 3. Solve the initial value problem in this case. Note that you have already found the eigenvectors and eigenvalues for the matrix which appears in this system, in problem 5.

(15 points)

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \\ 40 \end{bmatrix}.$$

$$\vec{x}_H(t) = c_1 \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix} + c_3 e^{-5t} \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

↑ #5

at $x=0$:

$$\left[\begin{array}{ccc|c} 6 & -2 & -1 & 40 \\ 1 & -1 & -1 & 20 \\ 3 & 3 & 2 & 40 \end{array}\right]$$

R_2, $-6R_2+R_1$, $-3R_2+R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 20 \\ 0 & 4 & 5 & -80 \\ 0 & 6 & 5 & -20 \end{array}\right]$$

$-R_2+R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 20 \\ 0 & -2 & 0 & -60 \\ 0 & 6 & 5 & -20 \end{array}\right]$$

$-6R_2+R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 20 \\ 0 & 1 & 0 & +30 \\ 0 & 0 & 5 & -200 \end{array}\right]$$

R_1+R_2

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & +50 \\ 0 & 1 & 0 & +30 \\ 0 & 0 & 1 & -40 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & -40 \end{array}\right]$$

so $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 30 \\ -40 \end{bmatrix}$

So, soltn is

$$\vec{x}(t) = 10 \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix} + 30 e^{-4t} \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix} - 40 e^{-5t} \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

6c) What happens to the salt amounts in each tank as t approaches infinity? Why would you get the same answer if the initial 100 pounds of salt were distributed in different proportions to the three tanks? Explain.

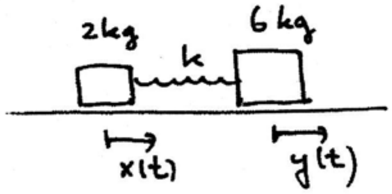
(5 points)

$$\lim_{t \to \infty} \vec{x}(t) = 10 \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 60 \\ 10 \\ 30 \end{bmatrix}$$

this will be the answer for any IVP (w 100 lbs total salt)
because limiting concentrations
will be equal in all 3 tanks,
so amounts will be proportional to tank volumes

10

7) Consider the following configuration of two masses held together with a single spring, on a frictionless table. The mass on the left is 2 kilograms, the one on the right is 6 kilograms. The Hooke's constant k is 6 Netwons/meter. Measure positive displacements from rest position to the right, as usual, and use x(t) and y(t) for these displacements, as indicated.



7a) Show that this mass-spring system satisfies the second order system of differential equations

$$\begin{bmatrix} x''(t) \\ y''(t) \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(5 points)

$2x'' = 6(y-x)$ so $x'' = -3x + 3y$

$6y'' = -6(y-x)$ $\quad y'' = x - y$

7b) Find the general solution to this system of differential equations. Hint: This is a second order system of two differential equations, so the solution space is 4-dimensional. You will need to get two linearly independent solutions from each of the matrix eigenspaces above. Think about what you are modeling to understand the solutions you can get from the eigenvector with zero eigenvalue.

(10 points)

$\vec{x}'' = Ax$

try for basis $\cos\omega t\,\vec{v}$, $\sin\omega t\,\vec{v}$ with $\omega^2 = -\lambda$, λ = eval of A.

$$|A - \lambda I| = \begin{vmatrix} -3-\lambda & 3 \\ 1 & -1-\lambda \end{vmatrix} = (\lambda+3)(\lambda+1) - 3 = \lambda^2 + 4\lambda = \lambda(\lambda+4)$$

$\lambda = 0$

$\left[\begin{array}{cc|c} -3 & 3 & 0 \\ 1 & -1 & 0 \end{array}\right]$

$\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = -4$, $\omega = 2$

$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 1 & 3 & 0 \end{array}\right]$

$\vec{v} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

$$\boxed{\vec{x}(t) = (c_1 + c_2 t)\begin{bmatrix} 1 \\ 1 \end{bmatrix} + (c_3\cos 2t + c_4 \sin 2t)\begin{bmatrix} -3 \\ 1 \end{bmatrix}}$$

either sitting still, or both masses moving at the same speed with $y - x = 0$

oscillating out of phase, mass 1 ampl. = 3 times mass 2 amp.

if $A\vec{v} = \vec{0}$

then $\vec{x}(t) = (c_1 + c_2 t)\vec{v}$ solves

$\vec{x}'' = 0 = (c_1 + c_2 t)A\vec{v}$!

8) Consider the system of differential equations below which models two populations x(t) and y(t). (You can think of this as an extension of problem (5) for the population x(t), which now finds itself in the presence of another species y(t).)

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 7x - x^2 - xy \\ -5y + xy \end{bmatrix}$$

8a) If this was a model of two interacting populations, which model would it be? Explain.

(2 points)

presense of y lowers x's growth rate

y needs x to have positive growth rate

prey-predator!

(prey is logistic w/o predator)

8b) Find the equilbrium solutions to this system of differential equations. There are ~~two~~ three! of them.

(3 points)

$7x - x^2 - xy = 0 = x(7-x-y)$

$-5y + xy = 0 = y(-5+x)$

$x = 0$ or $7-x-y=0$

$y=0$ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$y=0$ so $x=7$ $\begin{bmatrix} 7 \\ 0 \end{bmatrix}$

$x=5$, $y=2$ $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

8c) Only one of your equilbrium solutions has positive populations of both species. Linearize the population model near this equilbrium point. Use your analysis to classify which type of equilbrium point this is.

(10 points)

$\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$$J = \begin{bmatrix} F_x & F_y \\ G_x & G_y \end{bmatrix} = \begin{bmatrix} 7-2x-y & -x \\ y & -5+x \end{bmatrix}$$

Linearization

$$J(5,2) = \begin{bmatrix} -5 & -5 \\ 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} -5 & -5 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

$$|J - \lambda I| = \begin{vmatrix} -5-\lambda & -5 \\ 2 & -\lambda \end{vmatrix}$$

$$= \lambda^2 + 5\lambda + 10 = 0 \quad \lambda = \frac{-5 \pm \sqrt{25-40}}{2}$$

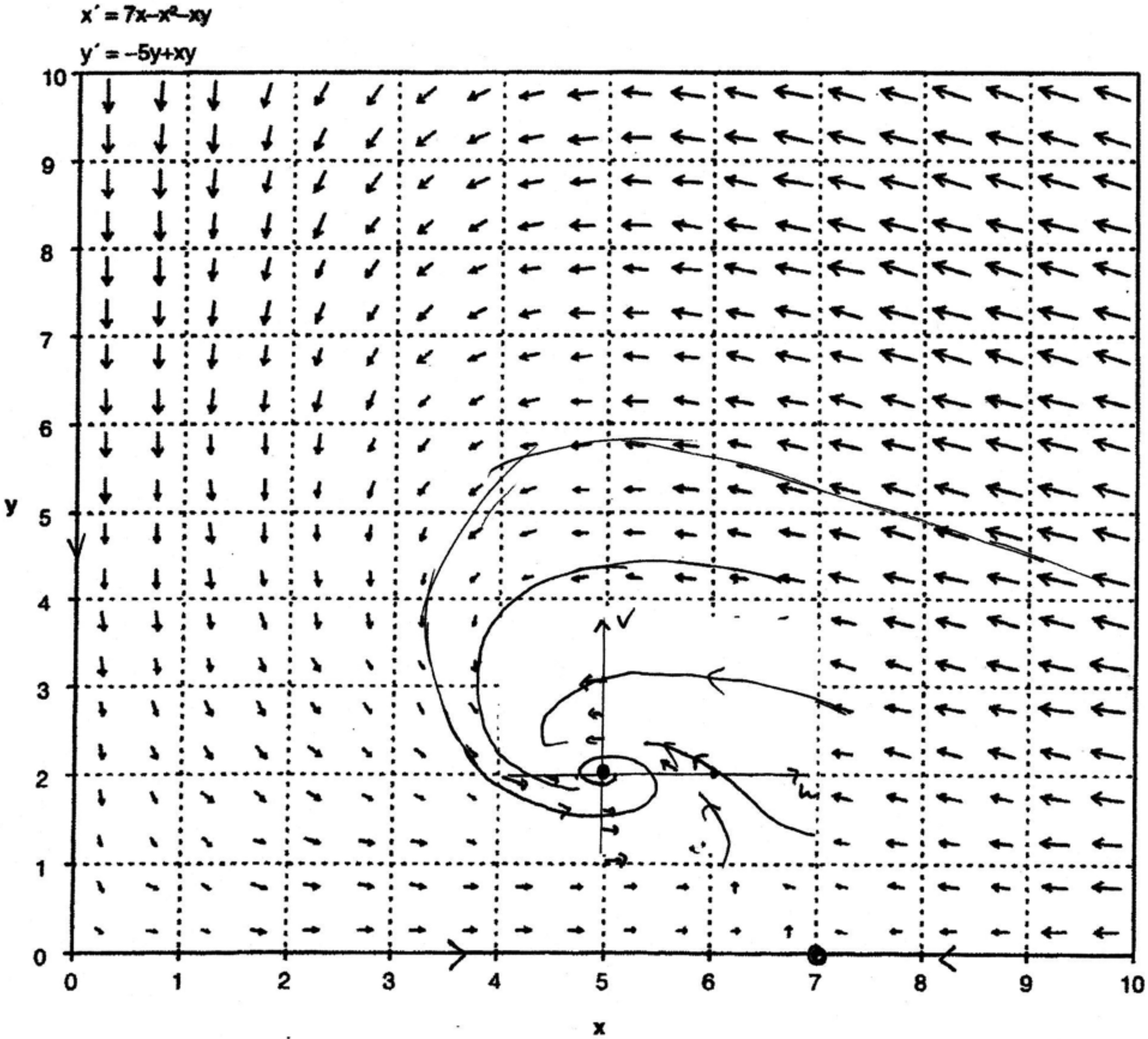
$$= -\frac{5}{2} \pm \frac{\sqrt{15}\, i}{2}$$

asymp. stable spiral

$$e^{\lambda t}\vec{v} = e^{-\frac{5}{2}t}\left(\cos\frac{\sqrt{15}}{2}t + i\sin\frac{\sqrt{15}}{2}t\right)\vec{v}$$

8d) Using your work from (8c), fill in the missing piece of the pplane phase portrait below. Also draw the the phase diagrams along each of the positive x and y axes. Then make a prediction about long term behavior of solutions to this population model, assuming both populations start out positive. (Depending on your analysis in (8c) your prediction may or may not depend on where in the first quadrant the initial population vector is located.)

(5 points)



I used $\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} -5 & -5 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$ to plot some tang vectors, then drew a rough sketch

It looks like all solns with pos. initial pops in both species converge to $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$.