

Name.....

Student I.D. number.....

Math 2250-1
Practice Exam
November 14, 2008

The exam is closed book and closed note, but a scientific calculator is allowed and may be useful. Calculators which can do linear algebra or solve differential equations are not allowed. In order to receive partial credit or full credit all work must be shown. There are 100 points possible on the test, and the point values of each problem are indicated in the right margin. You may wish to ration your time accordingly. GOOD LUCK!

1) Here is a matrix A:

$$A := \begin{bmatrix} 1 & 3 & 2 & 5 & 5 \\ -1 & 1 & 2 & -1 & 3 \\ 2 & 0 & -2 & 4 & -2 \\ -2 & 2 & 4 & -2 & 5 \end{bmatrix}$$

Here is its reduced row echelon form:

$$\text{RREF}(A) := \begin{bmatrix} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1a) Find all solutions to the homogeneous linear equation $Ax=0$.

(10 points)

1b) Exhibit a basis for the solution space you found in part 1a), and explain why (prove) it's a basis. Your explanation should make it clear that you know what it means for a collection of vectors to be a basis!

(5 points)

2) Consider the differential equation

$$y'''(x) - 2y''(x) + 5y'(x) = 0$$

2a) Find the general solution to this differential equation.

(10 points)

2b) Using your work from (2a) exhibit a basis for the solution space. Could you explain why your basis is a basis, if there had been room for more questions on this practice exam?

(5 points)

2c) Find the general solution to the nonhomogeneous linear differential equation

$$y'''(x) - 2y''(x) + 5y'(x) = 50x + 20e^{(2x)}$$

(10 points)

3) Consider this differential equation, which could arise as a mass-spring (or electrical circuit) problem.

$$x''(t) + c x'(t) + 3 x(t) = 0$$

3a) Identify possible values for mass, damping coefficient and Hooke's (spring) constant which could yield the differential equation above. Make sure to include correct units.

(6 points)

3b) Give the precise values or intervals of c which correspond to undamped, underdamped, critically damped, and overdamped problems, for the differential equation above.

(4 points)

3c) Now consider the forced oscillation problem,

$$x''(t) + c x'(t) + 3 x(t) = F_0 \cos(\omega t)$$

What values of " c " and " ω " would lead to resonance? What form would you guess for a particular solution, in this case? DO NOT ACTUALLY FIND THE PARTICULAR SOLUTION (because I have no points to spare on this practice exam).

(5 points)

3d) Now consider the damped forced oscillator

$$x''(t) + 4x'(t) + 3x(t) = -30 \sin(3t)$$

This differential equation has a particular solution

$$x_p(t) = 2 \cos(3t) + \sin(3t)$$

Use this particular solution along with the solution to the homogeneous problem (which you must find), in order to solve the initial value problem

$$x''(t) + 4x'(t) + 3x(t) = -30 \sin(3t)$$

$$x(0) = 2$$

$$x'(0) = -1$$

Identify the steady periodic and transient pieces of the solution.

(15 points)

4) Resolve the initial value problem in (3d) using Laplace transform. For your convenience, I repeat it here:

$$x''(t) + 4x'(t) + 3x(t) = -30 \sin(3t)$$

$$x(0) = 2$$

$$x'(0) = -1$$

(20 points)

5a) Find the function $x(t)$ which has Laplace transform

$$X(s) = \frac{2s + 1}{s^2 + 4s + 20}.$$

(5 points)

5b) Use the convolution definition to compute $(f*g)(t)$, for $f(t)=t$ and $g(t)=t^2$. Then use the Laplace transform table to check that your answer has Laplace transform which is the product of the Laplace transforms of $f(t)$ and $g(t)$.

(5 points)