Name..
I.D. number..

Math 2250-1

Practice Exam 1 -actual first exam from 2250-3 fall 2004

October 2008

This exam is closed-book and closed-note. You may use a scientific calculator, but not one which is capable of graphing or of solving differential or linear algebra equations. **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** There are 100 points possible. The point values for each problem are indicated in the right-hand margin.
Good Luck!

1) Consider a brine tank which holds 4000 gallons of continuously-mixed liquid. Let x(t) be the amount of salt (in pounds) in the tank at time t. The in-flow and out-flow rates are both 200 gallons/hour, and the concentration of salt flowing in is 0.02 pounds per gallon.
1a) Use the information above and your modeling ability to derive the differential equation for x(t):

$$\frac{dx}{dt} = 4 - \frac{1\,x}{20}$$

(5 points)

1b) What is the equilibrium solution to this differential equation? Is it stable or unstable? Explain.
(5 points)

1c) Find the solution to the initial value problem, assuming their are originally 20 pounds of salt in the tank.

(15 points)

1d) Is the limiting value of $x(t)$ as t approaches infinity consistent with your discussion in (1b)? How could your smart little sister have told you the limting amount of salt without understanding any differential equations, i.e. in terms of the in-flow concentration?

(5 points)

2) A certain population P(t) of animals satisfies a logistic growth differential equation, and this population is harvested at a constant rate, leading to the differential equation

$$\frac{dP}{dt} = -P^2 + 4P - 3$$

2a) Assume that the units of P(t) are thousands of animals, and that time is measured in years. At what rate are the animals being harvested?

(4 points)

2b) What are the equilibrium populations for this differential equation? Make a phase portrait and determine which equilibria are stable and which are unstable.

(8 points)

2c) If the initial population was 2000 animals ($P_0 = 2$), what do you expect the limiting population to be, based upon your work in (2b)?

(4 points)

2d) Solve the initial value problem for this harvested logistic equation, with $P_0 = 2$.

(15 points)

2e) Is your predicted limiting population from (2b) consistent with your actual solution from (2d)?

(4 points)

3) Consider the matrix A defined by

$$A := \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 3 & -6 \end{bmatrix}$$

3a) Compute the determinant of A.

(4 points)

3b) What does the value of the determinant you computed in (4a) tell you about whether or not the matrix A has an inverse? Explain, by quoting the general fact relating determinant values to matrix inverses.

(4 points)

3c) Find the inverse matrix to A, using the row-operation algorithm. Remember to show all work, as always. Hint: The correct inverse matrix has no fractions in it, which you could deduce from your work in the answer to part (3a)

(12 points)

3d) Recompute the inverse, using the adjoint formula. Show the cofactor matrix as well as the adjoint matrix as intermediate steps, so that I can check your work.

(10 points)

3e) Use the inverse matrix from part (3c) or (3d) to solve the system

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ -2 & 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -11 \end{bmatrix}$$

(5 points)