Math 2250-1
Wed October 8

4.1-4.3 HW due date extended to the Monday after break, Oct 20
(there will be additional 4.4, 4.5, 4.7 hw due that Friday Oct 24)

We finished thru page 3 of Tuesday's notes,
must now do pages 4, 5 there.

first, warm up by recalling linear algebra concepts:

linear combination of $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k$:

span $\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k\}$:

$\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k$ are linearly dependent :

linearly independent :

now do page 4-5 Tuesday, on vector spaces and subspaces

Exercise 1 Let $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \text{ s.t. } x+2y-3z=0 \right\}$ (you're used to saying "the plane $x+2y-3z=0$")

1a) Show W is a subspace by checking (α), (β), i.e. closure with respect to addition and scalar multiplication

(α)

(β)

1a') Is this a special case of the bullet point on page 5 Tuesday?
1b) Show W is a subspace by showing it is the span of 2 vectors in $\mathbb{R}^3$
hint: rref!

Exercise 2 Is the plane $x+2y-3z=1$ a subspace? Hint: no! Explain!

Exercise 3 span $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \right\}$ must be a plane thru $\vec{0}$, in $\mathbb{R}^3$

Find the equation $ax + by + cz = 0$ for this plane

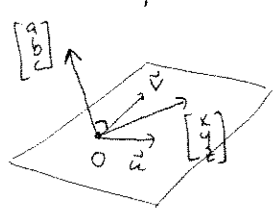
Ans: If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in W$ then we can solve

$$c_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 + 3c_2 \\ -c_1 \\ 2c_1 + c_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{for } c_1, c_2$$

$$\begin{array}{r|cc|c} & 1 & 3 & x \\ & -1 & 0 & y \\ & 2 & 1 & z \\ \hline -R_1 & 1 & 0 & -y \\ R_2 & 1 & 3 & x \\ & 2 & 1 & z \\ \hline & 1 & 0 & -y \\ -R_1+R_2 & 0 & 3 & x+y \\ -2R_1+R_3 & 0 & 1 & z+2y \\ \hline & 1 & 0 & -y \\ R_3 & 0 & 1 & z+2y \\ R_2 & 0 & 3 & x+y \\ \hline & 1 & 0 & -y \\ & 0 & 1 & z+2y \\ -3R_2+R_3 & 0 & 0 & x+y-3(z+2y) = x-5y-3z \end{array}$$

so $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in W$ iff $\boxed{x - 5y - 3z = 0}$

another way for this special case:



$ax + by + cz = 0$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & -1 & 2 \\ 3 & 0 & 1 \end{vmatrix} = (-1, 5, 3)$$

$-x + 5y + 3z = 0$ ✓

<u>example</u> Describe all subspaces of $\mathbb{R}^3$:

Let W be such a subspace

(0) $\vec{0} \in W$.

that might be all!

if W has more elements, let $\vec{u} \in W$, $\vec{u} \neq \vec{0}$

then $\underset{\substack{\uparrow \\ \text{"is a subset of"}}}{\operatorname{span}(\vec{u}) \subset W}$

(1) that might be all. In this case $W = \{ t\vec{u},\ t \in \mathbb{R} \}$ is a <u>line thru $\vec{0}$</u>

if W has more elts, let $\vec{v} \in W$, $\vec{v} \notin \operatorname{span} \vec{u}$.

then $\{\vec{u}, \vec{v}\}$ are linearly independent: (if $c_1\vec{u} + c_2\vec{v} = \vec{0}$, $c_2 \neq 0 \Rightarrow \vec{v} \in \operatorname{span} \vec{u}$, so $c_2 = 0$. then $\vec{u} \neq 0 \Rightarrow c_1 = 0$)

and $\operatorname{span}\{\vec{u}, \vec{v}\} \subset W$.

(2) that might be all. In this case $W = \{ t\vec{u} + s\vec{v},\ s, t \in \mathbb{R} \}$

is a plane.

See last example;

$$\left[\begin{array}{c|c|c} \vec{u} & \vec{v} & \begin{matrix} x \\ y \\ z \end{matrix} \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & \sim \\ 0 & 1 & \sim \\ 0 & 0 & ax+by+cz \end{array}\right]$$

if that's not all, then let $\vec{w} \in W$, $\vec{w} \notin \operatorname{span}\{\vec{u}, \vec{v}\}$.

Then

$$\left[\begin{array}{c|c|c} u & v & w \end{array}\right] \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(otherwise ~~either $\vec{v}$ and $\vec{u}$ are dependent, or~~ $\vec{w}$ is a l.c. of $\vec{u}, \vec{v}$).

Thus $\operatorname{span}\{\vec{u}, \vec{v}, \vec{w}\} = \mathbb{R}^3$

(3). So $W = \mathbb{R}^3$

<u>Def</u> A <u>basis</u> for the vector space V is a linearly independent set $\{\vec{v}_1, \dots \vec{v}_k\}$ in V which also spans V

<u>Def</u> The number of elements in a (any!) basis for V is the <u>dimension</u> of V.

We exhibited the dimension $0, 1, 2, 3$ subspaces of $\mathbb{R}^3$.