Math 2250-1
Friday Oct 3

§ 4.1–4.3 cont'd.

HW for Friday Oct 10
4.1 1, 7 9 10 15, 16, 19, 22, 25, 26, 31, 33
4.2 5, 6, 9, 15, 18 24, 27 29
4.3 1, 3, 6, 9, 10 16 17 18 23 25
Please feel free to use technology to compute rref in any of this week's problems.

Exam 1 on Monday!
Problem session tomorrow (Saturday)
10:30 a.m – noon
LCB 219
(practice test is posted.)

Recall

Chapter 3

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n \end{bmatrix}$$

matrix form

$$= x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

Chapter 4

linear combination form

the language we will use:

If $\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k\}$ is a collection of vectors (say in $\mathbb{R}^n$) then $\vec{w}$ is a linear combination of $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k$ means

$\vec{w} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k$ for some choice of the scalar (numbers) $c_1, c_2, \ldots c_n$. These numbers are called the linear combination coefficients

the span of $\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k\}$ is the collection of all $\vec{w}$'s which are linear combinations of $\vec{v}_1, \ldots \vec{v}_k$.

Example On Wednesday we showed that $\begin{bmatrix} 1 \\ 8 \end{bmatrix}$ is in the span of $\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ since $\begin{bmatrix} 1 \\ 8 \end{bmatrix} = -3\begin{bmatrix} 3 \\ -2 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 1 \end{bmatrix}$

Exercise 1:
- Is every vector in $\mathbb{R}^2$ a linear combination of $\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}$?
- What is the span of $\left\{\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}\right\}$?
- If $s\begin{bmatrix} 3 \\ -2 \end{bmatrix} + t\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, are the linear combination coefficients unique?

(this replaces page 5 Wed.)

Notation: $\hat{i} = \vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{j} = \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \vec{e}_3$.

these 3 vectors are often called the <u>standard basis</u> of $\mathbb{R}^3$

<u>Exercise 2</u> a) Express $\begin{bmatrix} 3 \\ 8 \\ -5 \end{bmatrix}$ as a linear combination of $\vec{e}_1, \vec{e}_2, \vec{e}_3$.

b) Can every vector in $\mathbb{R}^3$ be expressed as a linear combo of $\vec{e}_1, \vec{e}_2, \vec{e}_3$?
Are the linear combo coeff's unique?

c) What is the span of $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$

d) What is the span of $\{\vec{e}_1, \vec{e}_2\}$?

e) What is the span of $\{\vec{e}_1\}$?

Exercise 3 Let $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -5 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 4 \\ -1 \\ 11 \end{bmatrix}$

3a) Is $\begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix}$ a linear combination of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

You might make use of the following Maple output:

```
> with(linalg):
> A:=matrix(3,3,[2,-1,4,1,1,-1,1,-5,11]):
  b:=vector([6,3,3]):
  Aaugb:=augment(A,b);
  rref(Aaugb);
```

$$Aaugb := \begin{bmatrix} 2 & -1 & 4 & 6 \\ 1 & 1 & -1 & 3 \\ 1 & -5 & 11 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

i.e. find c_1, c_2, c_3 so that

$$c_1 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -5 \end{bmatrix} + c_3 \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix}$$

3b) Are the linear combo coeff's in 3a) unique?

3c) Explain why $\text{span}\{\vec{v}_1, \vec{v}_2\} = \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. What geometric object is this span?

Hint: $\left[\begin{array}{ccc|c} 2 & -1 & 4 & x \\ 1 & 1 & -1 & y \\ 1 & -5 & 11 & z \end{array}\right] \xrightarrow[\text{(not rref)}]{\text{reduce}} \left[\begin{array}{ccc|c} 1 & 1 & -1 & y \\ 0 & 3 & -6 & -x+2y \\ 0 & 0 & 0 & -2x+3y+z \end{array}\right]$

Exercise 4 If you have 3 vectors, $\vec{v}_1, \vec{v}_2, \vec{v}_3$ in $\mathbb{R}^3$, what matrix condition is equivalent to:
$\text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \mathbb{R}^3$, and that the linear combo coeff's are unique?
(If this is the case, we call $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ a basis for $\mathbb{R}^3$.)