Math 2250-1
Tuesday Nov 4
Experiment Day!

Experiment notes
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Pendulum:

```
> restart:
> Digits:=5:
```

Here is my measurement of L, which we'll check again in class. It (correctly) assumes the effective length of the pendulum uses the the distance to the ball's center of mass, from the top. (But, where exactly is the top?!). Wikipedia says the acceleration of gravity is about

```
> g:=9.780327*(1+.00530*(sin(phi))^2)-3.086*h*10^(-6);
```

$$g := 9.780327 + 0.051836 \sin(\phi)^2 - 0.30860\ 10^{-5}\ h$$

at latitude phi and height h meters

```
> phi:=(40+47.0/60)*(evalf(Pi))/180.;  #SLC latitude
  h:=4500.*.3048;  #UU elevation?
  g;  #looks pretty close to 9.80 here!
```

$$\phi := 0.71179$$
$$h := 1371.6$$
$$9.7982$$

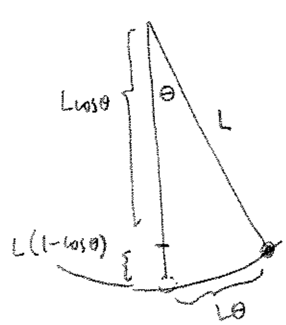
Prediction (this may change if we re-measure!)

```
> L:=1.530;
  omega:=sqrt(g/L); #radians per second
  f:=evalf(omega/(2*Pi)); #cycles per second
  T:=1/f; #seconds per cycle
```

$$L := 1.530$$
$$\omega := 2.5306$$
$$f := 0.40276$$
$$T := 2.4829$$

What do we get when we do the experiment???

Error sources?



$$KE + PE = \tfrac{1}{2} m v^2 + mgL(1-\cos\theta) \qquad v = L\theta'$$

$$0 = D_t(KE+PE)$$

$$= \tfrac{1}{2} m L^2\, 2\theta'\theta'' + mgL\sin\theta\,\theta'$$

$$0 = mL\theta'\left[L\theta'' + g\sin\theta\right]$$

$$L\theta'' + g\theta \approx 0$$

$$\omega_0 = \sqrt{\tfrac{g}{L}}$$

Mass-spring: (we might re-measure.)
How to find Hookes constant:

```
> 51-7.6; #add 50g and measure
      #displacement
```

$$43.4$$

```
> m:=.1;  #the mass is 100g = 0.1 kg
```

$$m := 0.1$$

My measurement for k, using a 50g mass and measuring the displacement in meters

```
> solve(k*.434=.05*g,k);
```

$$1.1288$$

```
> k := 1.1288;
```

$$k := 1.1288$$

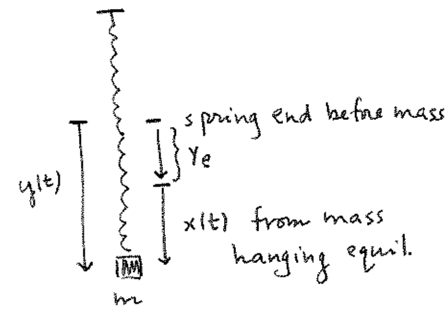
```
> omega:=sqrt(k/m);#radians per second
  f:=omega/evalf(2*Pi); #cycles per second
  T:=1/f; #seconds per cycle
```

$$\omega := 3.3598$$

$$f := 0.53473$$

$$T := 1.8701$$

What do we get in our experiment?



with gravity:

$my'' = -ky + mg$

$my'' + ky = mg$

$y = \frac{mg}{k} + y_H$

$y = y_e + y_H$

hidden gravity

$mx'' = \underline{\text{net}}\text{ force} = -kx$

$mx'' + kx = 0$

$y = \frac{mg}{k} + x(t).$

$\omega_0 = \sqrt{\frac{k}{m}}$

Correction, to account for spring KE, since it has mass!

```
> ms:=.011: #spring has mass 11g
  M:=m+ms/3; #"effective mass"
             #of spring is 1/3 actual mass
```

$$M := 0.10367$$

```
> omega1:=sqrt(k/M); #new angular freq est.
  f1:=omega1/evalf(2*Pi); #new freq est.
  T1:=1/f1;#new period est
```

$$\omega 1 := 3.2998$$

$$f1 := 0.52518$$

$$T1 := 1.9041$$

short story:

$$\boxed{\begin{array}{l}\text{governing equation is} \\ M x'' + k x = 0 \\ M = m + \tfrac{1}{3} m_s\end{array}}$$

long story:

If spring has mass m_s:

$$KE = \tfrac{1}{2} m (y')^2 + KE_{spring}$$

$$= \tfrac{1}{2} m (y')^2 + \tfrac{1}{2}\,\tfrac{1}{3} m_s (y')^2$$

$$\boxed{KE = \tfrac{1}{2} M (y')^2} \quad \boxed{M = m + \tfrac{1}{3} m_s}$$

KE_{spring} computation:

|| an integral!

$$\int_0^1 \tfrac{1}{2} \underbrace{(\mu y')^2}_{v^2} \underbrace{(m_s\, d\mu)}_{\text{mass}}$$

$$= \tfrac{1}{2}(y')^2 m_s \underbrace{\int_0^1 \mu^2\, d\mu}_{\frac{1}{3}}$$

μ is fraction of the way from fixed top to moving mass

$$PE = -mgy - m_s g \frac{y}{2} + \tfrac{1}{2} k y^2$$

how far ctr of mass of spring moves

stretch energy.

$$\text{Energy} = KE + PE$$

$$= \tfrac{1}{2} M (y')^2 + \tfrac{1}{2} k y^2 - C y$$

$$C = (g)(m + \tfrac{m_s}{2})$$

$$0 \equiv D_t E = M y' y'' + k y y' - C y'$$

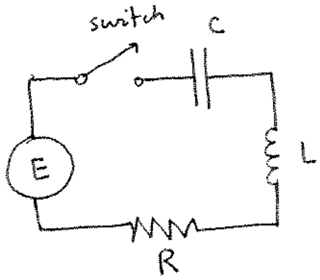
$$= y' \left[M y'' + k y - C \right]$$

$$M y'' + k y = C$$

$$y = \frac{k}{C} + y_H$$

$$\boxed{\omega_0 = \sqrt{\frac{k}{M}} = \sqrt{\frac{k}{m + \frac{1}{3} m_s}}}$$

RLC circuits. EP 3.7



voltage E volts (electromotive force)

Circuit elt	Voltage drop	units	
Inductor	$L \frac{dI}{dt}$	L henries	H
Resistor	RI	R ohms	Ω
Capacitor	$\frac{1}{C} Q$	C farads	F

$Q(t)$ = charge in ~~coto~~ coulombs (resides on capacitor)

$I(t) = Q'(t)$ = current (amperes) moving around circuit

Ohms Law (like Newton's)

the sum of the voltage drops around a circuit equals the applied voltage E

$$\boxed{\begin{aligned} &LQ'' + RQ' + \tfrac{1}{C}Q = E(t) \\ D_t: \quad &LI'' + RI' + \tfrac{1}{C}I = E'(t) \end{aligned}}$$

mathematically identical to

$$\begin{array}{cccc} mx'' & + cx' & + kx & = F(t) \\ \updownarrow & \updownarrow & \updownarrow & \\ L & R & 1/C & \end{array}$$

Exercise 1 Set up an IVP for a circuit in which

$R = 16\,\Omega$
$L = 2\,H$
$C = .02\,F$
$E(t) = 100V$
$I(0) = 0$
$Q(0) = 5$

Set up one for $Q(t)$ and one for $I(t)$

Example 2 (old) radios — resonance can be good if you're an electrical engineer.

translate!

Recall,
Our hand work for damped forced oscillator

$$mx'' + cx' + kx = F_0 \cos\omega t$$

$$x_p = x_{sp}(t) = A\cos\omega t + B\sin\omega t$$

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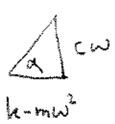
$$= C\cos(\omega t - \alpha)$$

$$A = F_0\left(\frac{k-m\omega^2}{(k-m\omega^2)^2 + c^2\omega^2}\right)$$

$$B = F_0 \frac{c\omega}{(k-m\omega^2)^2 + c^2\omega^2}$$

so $C = \dfrac{F_0}{\sqrt{(k-m\omega^2)^2 + c^2\omega^2}}$



$0 < \alpha < \pi$

got practical resonance with $\omega \approx \omega_0$, $c \approx 0$.

note.

$$LQ'' + RQ' + \frac{1}{C}Q = E_0 \sin\omega t$$

D_t: $\quad LI'' + RI' + \frac{1}{C}I = E_0\omega \cos\omega t$

steal left column.

$$I_{sp}(t) = I_0 \cos(\omega t - \alpha)$$

$$I_0 = \frac{E_0}{\sqrt{(\frac{1}{\omega C} - L\omega)^2 + R^2}}$$

max response (for fixed E_0, R, ω)
when $\frac{1}{\omega C} = L\omega$

$\boxed{C = \frac{1}{L\omega^2}}$, $I_0 = \frac{E_0}{R}$

old (crystal) radios used the knob to adjust the capacitance to amplify the radio wave!

See p. 230-231 for more