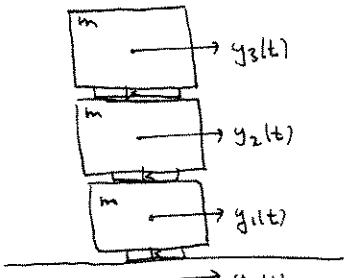
Earthquake Project Comments

(A careful explanation of the book's claim that even though the building is only being shaken at the base, in a frame of reference moving with ground, each floor experiences an equal "inertial force".)

consider a 3 story building (in project is 7 stories) modeled as a mass-spring model, with each story having mass m, and floor junctions modeled as "springs" with constants k (with respect to transverse as opposed to parallel motion)



$y_0(t) = E\cos\omega t$ → transverse (shaking) wave

$E\cos\omega t$

$y_0(t) = E\cos\omega t$ so

$y_0'' = -E\omega^2\cos\omega t$

$m y_1'' = -k(y_1 - y_0) + k(y_1 - y_2)$

$m y_2'' = -k(y_2 - y_1) - k(y_2 - y_3)$

$m y_3'' = -k(y_3 - y_2)$

reduction of fns

let

$x_0 = y_0 - E\cos\omega t \equiv 0$

$x_1 = y_1 - E\cos\omega t$

$x_2 = y_2 - E\cos\omega t$

$x_3 = y_3 - E\cos\omega t$

$x_0 \equiv 0$

$m x_1'' = m y_1'' + mE\omega^2\cos\omega t$

$m x_2'' = m y_2'' + mE\omega^2\cos\omega t$

$m x_3'' = m y_3'' + mE\omega^2\cos\omega t$

so

$m x_1'' = -k(x_1 - 0) - k(x_1 - x_2) + mE\omega^2\cos\omega t$

$m x_2'' = -k(x_2 - x_1) - k(x_2 - x_3) + mE\omega^2\cos\omega t$

$m x_3'' = -k(x_3 - x_2) + mE\omega^2\cos\omega t$

so the x's are the displacements from equilibrium as measured by an observer on the ground

Notice $y_i - y_j = x_i - x_j$

$$\begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} = \begin{bmatrix} -2k/m & k/m & 0 \\ k/m & -2k/m & k/m \\ 0 & k/m & -k/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + E\omega^2\cos\omega t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

it's as if all floors are being forced - book calls this inertial forcing due to frame of reference