

MATH 2250-1
Mass-Spring systems
December 1, 2008

This maple document covers section 7.4 and contains commands you will need for the Earthquake exploration you're doing at the end of that section. Here is how Maple computations work out the running example in today's notes.

```
[ > with(linalg):
> with(plots):with(DEtools):
> M:=matrix([[2,0],[0,1]]);
  K:=matrix([[-150,50],[50,-50]]);
  A:=evalm(inverse(M)*K);

                M :=  $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ 
                K :=  $\begin{bmatrix} -150 & 50 \\ 50 & -50 \end{bmatrix}$ 
                A :=  $\begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}$ 

> eigenvectors(A);

[-100, 1, {[ -1, 1 ]}, [-25, 1, {[ 1, 2 ]}]
```

Therefore, the natural frequencies of this system are the 10 and 5, and the two fundamental modes correspond to the masses moving in opposite directions (with equal amplitudes and angular frequency 10) and in parallel directions (with amplitude ratio of two and angular frequency 5), as discussed on page 2 of today's handwritten notes.

Now, let's consider the forced system with force vector equal to $\cos(\omega t)[0,50]$, i.e. the second mass is being forced periodically. In other words, the system $M\mathbf{x}'' = K\mathbf{x} + \mathbf{F}$, where $\mathbf{F} = \cos(\omega t)[0,50]$; this is Example 3 on page 327, and worked out in today's notes.

```
[ > F0:=evalm(inverse(M)*vector([0,50])):
    #The F0 in the normalized equation (30), page 436...
    #needs serious modification for Maple project
  Iden:=array(1..2,1..2,identity):
    #the 2 by 2 identity matrix
  Aleft:=omega->evalm(A + omega^2*Iden):
    #the matrix function multiplying
    #c on the left side of (32)
  c:=omega->evalm(-inverse(Aleft(omega))*F0):
    #the solution vector c(omega) to (32),
    #obtained by multiplying both sides of equation
    #(32) on the left, by the inverse to Aleft
> c(omega); #see equation (35) page 437,
    #and our hand work on page 3 of today's notes
```

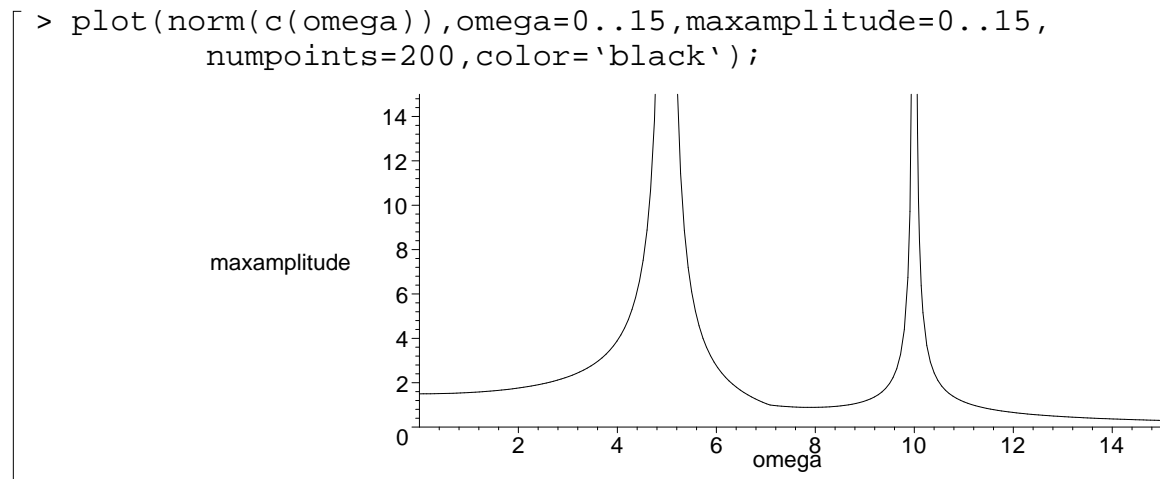
$$\left[\frac{1250}{2500 - 125\omega^2 + \omega^4}, -\frac{50(-75 + \omega^2)}{2500 - 125\omega^2 + \omega^4} \right]$$

The vector $c(w)$, as above, times the function of time, $\cos(wt)$, is a particular solution to the forced oscillation problem we are considering. If we assume that our actual problem has a small amount of damping, then we expect that this particular solution is very close to the steady periodic solution to the damped problem, as we just discussed and also on pages 437-438 text. Study resonance phenomena for these slightly damped problems by plotting the maximum amplitude for the individual masses, in the steady state solutions to the undamped problems. Maple has the command “norm” to measure this maximum amplitude, ready for you to use in your Maple project.

```
> norm(c(omega));
```

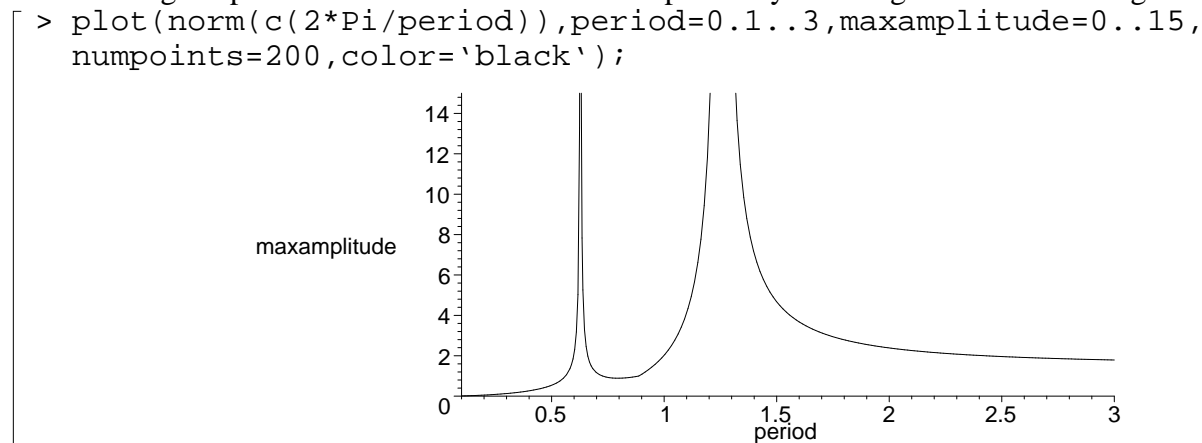
$$\max\left(\left|\frac{1250}{2500 - 125\omega^2 + \omega^4}\right|, 50\left|\frac{-75 + \omega^2}{2500 - 125\omega^2 + \omega^4}\right|\right)$$

The following graph of the maximum amplitude norm for the undamped particular solution shows that in the slightly damped problem we expect practical resonance when ω is near 5 or 10 radians per second:



This is qualitatively the picture on page 437, figure 7.4.10, although they plotted the (Euclidean) magnitude of $c(\omega)$ rather than the maximum of the individual amplitudes. Notice how we get Maple to label the axes as desired

We can get a plot of resonance as a function of period by recalling that $2\pi/T = \omega$:



COMMENTS FOR THE EARTHQUAKE PROJECT:

(1) Students are often confused by the forcing term in equation (2) of page 440, namely

$$\begin{aligned} & > E * (\omega)^2 * \cos(\omega * t) * b; \\ & \qquad \qquad \qquad E \omega^2 \cos(\omega t) b \end{aligned}$$

where b is the transpose of $[1,1,1,1,1,1,1]$. They ask, “how can the earthquake be forcing all seven stories, it seems like it’s just shaking the bottom one.” Well, the students are correct, but so is Edwards-Penney. The authors talk about an “opposite inertial force” being the reason for this forcing term and there’s a detailed discussion in some notes I’ve posted with this project on our project page. Here’s a brief summary. Think of the ground as the zeroth story. In the rest frame it is shaking with oscillation $E \cos(\omega t)$. And so its acceleration is its second time derivative, namely $-E \omega^2 \cos(\omega t)$. If you write down the inhomogeneous system of EIGHT second order DE’s for the accelerations of stories zero thru seven, the forcing (well, accelerating) term is $-E \omega^2 \cos(\omega t) [1,0,0,0,0,0,0]$, as you would expect. Call the solution 8-vector to this system $y(t)$, then see what the shaking looks like to someone on the ground by letting $x(t) = y(t) - E \cos(\omega t) [1,1,1,1,1,1,1]$. Then the zeroth story component of $x(t)$ will be identically zero, and the other seven components will satisfy equation (2) on 440, exactly as the authors claim. This is worked out in detail in the notes at

<http://www.math.utah.edu/~korevaar/earthquakecomments.pdf>

Very important note (also repeated on your Maple project).

(2) For large matrices the eigenvect command won’t work well unless you enter at least one decimal number; if all entries are rational numbers (expressed without decimal points), Maple tries to find the eigenvalues and eigenvectors algebraically and exactly, instead of numerically, and often fails. **Make sure at least one of your matrix entries has a decimal point in it.**