Math 2250-1

MAPLE

1.3-1.4: separable DE's, slope fields, existence and uniqueness.

Friday August 29, 2008

Homework for Friday September 5: (Hand in underlined problems.)

Maple Introduction for Math 2250, linked from our home page. The purpose of this assignment is to introduce you to Maple, its help features, some useful commands, and your ability to use it in order to make documents like these class notes. I highly recommend you complete this assignment, which is to read and work through the notes for the Introduction to Maple sessions we are holding for all sections of Math 2250. (This work will not be graded.)

1.3: **3, 8, 21, 24**. Use the the applet "dfield" for **8** and **24**. (google it. If you're using Math Department system the on-line applet won't print hard copies, but we've made a copy of the applet you can call from a terminal window which will print hard copies. This copy is also called dfield.). Also do problems 11, **12, 13, 14**.
1.4: 3, **4, 9, 12**, 14, 19, **20, 22, 40**, 45, **49, 54.**
1.5: 1, 7, **8, 13, 20**, 33, **36, 38, 41**.

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I created today's notes using the software package Maple, to show you the kind of documents it is possible to create with Maple. Although this software has been produced by a private company since 1988, it originated with Canadian government support at the University of Waterloo; the originators used the leaf on the Canadian flag to motivate the name "Maple."

You will be using Maple in Math 2250, in order to solve problems and do computer projects. You should think of this software as analgous to the service provided by Microsoft Word as a tool for writing papers - except Maple lets you create documents which combine text and mathematics. Alternately, you may use Maple to generate mathematical output, which you can then export to other documents. (That is closer to how engineers use Matlab, which is more efficient than Maple at doing extremely large computations, but lacks Maple's ability to create text documents and to easily do symbolic mathematical work.)

We will use Maple 8 in Math 2250, because it has superior graphing capabilites over the more recent versions, which are Maple 11 and Maple 12. The main advantage of these latter versions is that they include more Microsoft Word type bells and whistles. Any Maple 8 document should open in Maple 11-12, but if you are using these latter Maple versions and you expect to reopen your document in Maple 8, you must save your file in the "classic" mode. Student computers around campus (Engineering, Marriott, Math Department, Heritage Commons) have Maple software installed, and in most places you can find both the earlier and latter versions. The University bookstore sells a student version of Maple 12 ($130), which you may wish to purchase at some point, although it is not at all necessary to do so for Math 2250.

When you open any of these Maple versions you can find a "new user's tour" in a "help" window, and I recommend you take this tour to get an overview of Maple's capabilities. From then on, you will learn to use Maple the same way you learned to use Microsoft Word, i.e. by using it, using the help features, and asking friends, lab assistants, TAs and teachers when you get stuck. We will also be running Maple introductions sessions specifically for this course, Math 2250. The currently scheduled sessions (we may add more) are:

Saturday August 30: PCLab 1735, in Marriott MMC, 2-3 pm.
Tuesday September 2: LCB 115, 11:50-12:40 and 2:30-3:20
Wednesday September 3: LCB 115, 10:45-11:55.

These sessions are first-come, first-serve, and about 30 people can take a session at a time.

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We will use today's notes to continue our discussion in Wednesday's notes of slope fields and of the existence-uniqueness theorem, section 1.3. We will enhance that discussion by reminding ourselves how to solve separable differential equations, which is the main topic of section 1.4:

Separable Differential Equations

A first order differential equation

$$\frac{dy}{dx} = \mathrm{h}(x, y)$$

is called **separable** iff h(x,y) is a product of a function of x times a function of y,

$$\frac{dy}{dx} = \mathrm{g}(x)\,\phi(y)\,.$$

This is equivalent to the DE

$$\frac{dy}{dx} = \frac{\mathrm{g}(x)}{\mathrm{f}(y)}\ ,$$

where f and ϕ are reciprocal functions.

Solving separable DEs:

The algorithm is very simple, but magic: treat dy/dx as a quotient of differentials (?!), and multiply through to rewrite the DE as

$$\mathrm{f}(y)\,dy = \mathrm{g}(x)\,dx\ .$$

Then antidifferentiate the left side with respect to y and the right side with respect to x:

$$\int \mathrm{f}(y)\,dy = \int \mathrm{g}(x)\,dx\ .$$

If F(y) and G(x) are antiderivatives of f(y) and g(x), respectively, and if you collect the constants of integration on one side of the equation, then this yields an equation involving the solution function y, and the variable x:

$$\mathrm{F}(y) = \mathrm{G}(x) + C\ .$$

This equation defines y implicitly as a function of x. Sometimes you can use algebra to explicitly solve for y. The constant C can be adjusted to solve initial value problems.

<u>Mathematical justification for the method of separating differentials:</u>

The use of differentials is disguising an application of the chain rule. Here is the explanation for the magic method: The differential equation

$$\frac{dy}{dx} = \frac{\mathrm{g}(x)}{\mathrm{f}(y)}$$

can be rewritten without differential magic, as

$$\mathrm{f}(y)\left[\frac{dy}{dx}\right] = \mathrm{g}(x).$$

If y(x) is any solution to this rewritten equation, then the left side, namely

$$\mathrm{f}(\mathrm{y}(x))\left[\frac{dy}{dx}\right]$$

is the derivative with respect to x of

$$\mathrm{F}(\mathrm{y}(x)) \, ,$$

where F(y) is any antiderivative of f(y) (with respect to y). This is just the chain rule! Thus if G(x) is any antiderivative of g(x) (w.r.t.x), we can legally antidifferentiate the rewritten DE with respect to x (on both sides) to get

$$\mathrm{F}(\mathrm{y}(x)) = \mathrm{G}(x) + C$$

which is what we got by differential magic before!

We can use separation of variables to derive the solutions related to the slope field existence-uniqueness discussions on Wednesday's notes....so make sure you have a copy of those handy.

Exercise 4, page 3 Wednesday Aug 29 notes: This exercise asked you to use a slope field analysis for a model of a dropped ball whose downward velocity satisfies the initial value problem

$$\frac{dv}{dt} = 32 - 0.16\, v$$

$$\mathrm{v}(0) = v_0$$

to guess at what speed the ball hits the ground. Complete 4a), 4b) on Wednesday's notes, and then use separation of variables to find the precise function formulas for solutions to the IVP.

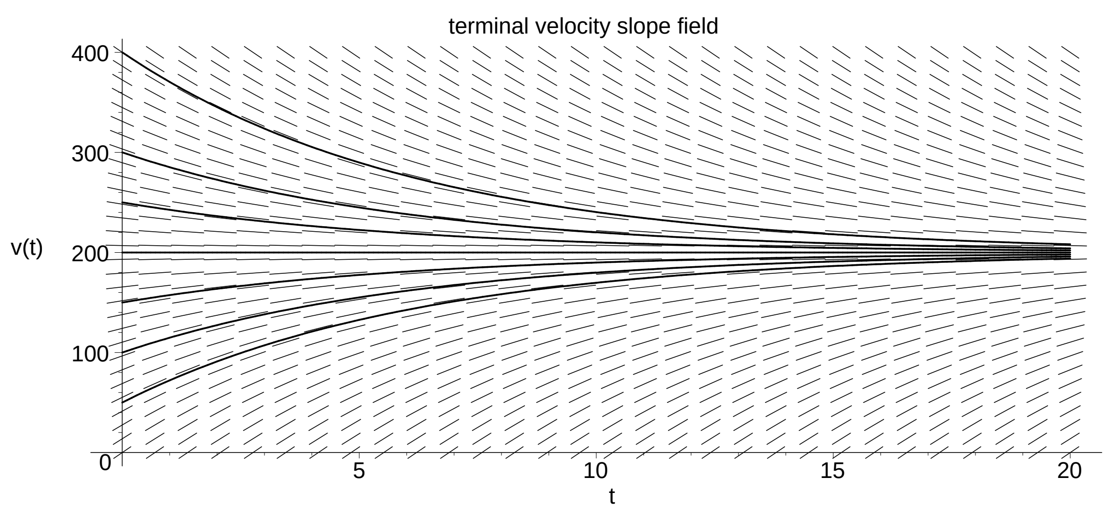
Here's how you can use Maple to check your answer (once you find the right commands):

```
> with(DEtools):  #a package of differential equation commands
> dsolve([diff(v(t),t)=32-.16*v(t),v(0)=v[0]]);
```

$$\mathrm{v}(t) = 200 + \mathbf{e}^{\left(-\frac{4\,t}{25}\right)}(-200 + v_0)$$

Maple can also draw slope fields and solution graphs, although the commands are more cumbersome than in the applet "dfield":

```
> deqtn:=diff(v(t),t)=32-.16*v(t):  #this is the DE
  DEplot(deqtn,v(t),t=0..20,{[v(0)=50],[v(0)=100],
     [v(0)=150],[v(0)=200],[v(0)=250],[v(0)=300],[v(0)=400]},
     v=0..400,arrows=line, color=black,linecolor=black,
     dirgrid=[30,30],stepsize=.1, title='terminal velocity slope
  field');
```



Discuss the existence and uniqueness theorem on page 4 of Wednesday's notes. Then do Exercises 5 and 6 in Wednesday's notes, and find solutions to these separable DEs whenever possible, to add insight to your existence-uniqueness discussion.