Math 2250
Wed Aug 27 §1.2-1.3

- swimmer problem, page 2 Tuesday notes
- recovering position from velocity example, page 3 Tuesday notes.

// that concludes §1.1-1.2 planned discussions. Any ?'s related to Fri HW? (§1.1-1.2)

problem sessions tomorrow:

7:30-8:20	WEB 108
8:35-9:25	JFB 103
12:55-1:45	JTB 130

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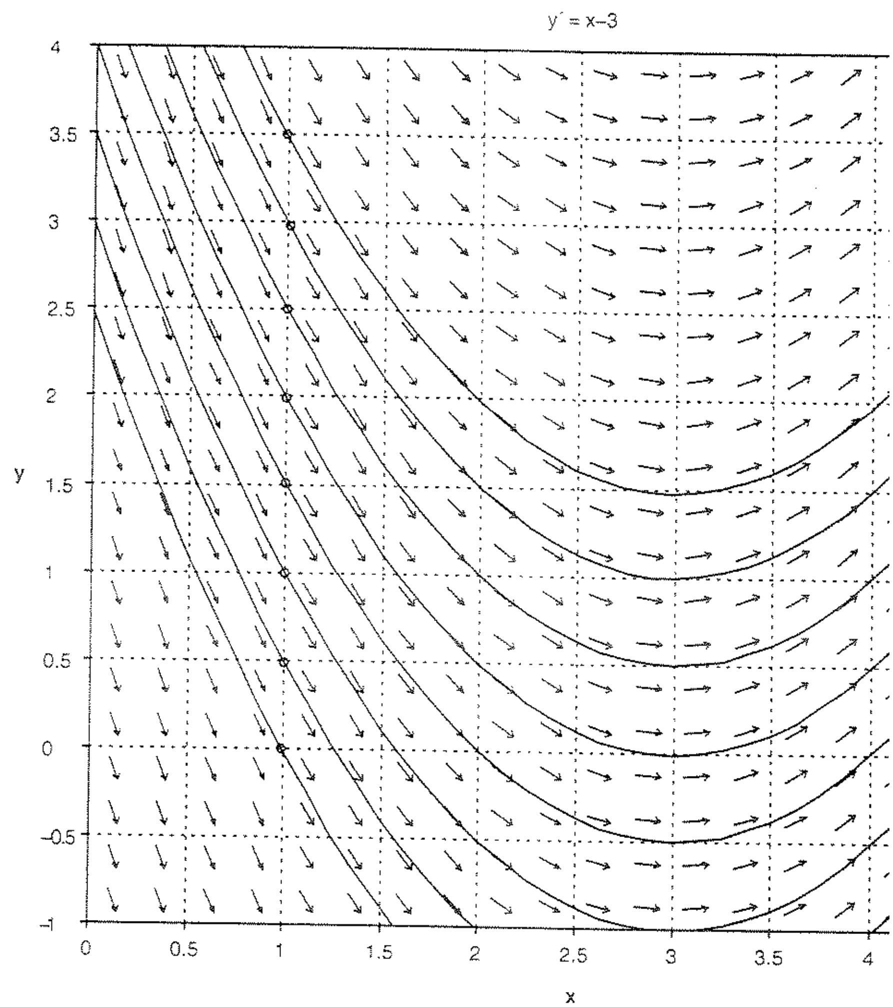
§1.3: slope fields, existence (∃) & uniqueness (!) for solutions to IVPs.

<u>Slope fields</u>: Yesterday we solved the IVP

(1) IVP $\begin{cases} \frac{dy}{dx} = x-3 \\ y(1) = 2 \end{cases}$

by antidifferentiation.
The solution was $y(x) = \frac{x^2}{2} - 3x + \frac{9}{2}$

We then used a <u>slope field</u> (and <u>isoclines</u>) to deduce what the graph of the sol'n looked like, <u>independently</u> of the formula for the solution.



google "dfield" (stands for direction field) and you will be led to a java applet (located at URL http://math.rice.edu/~dfield/dfpp.html) to draw pictures as at right. Do this!!

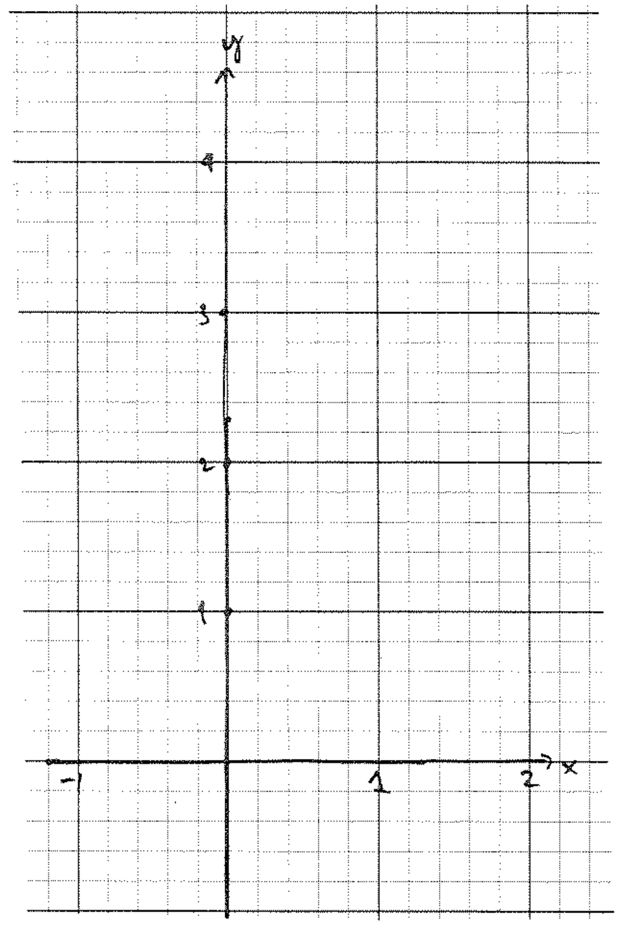
<u>Exercise 1</u> Which of the curves sketched above is the graph of the solution to IVP (1)?

Exercise 2 Sketch the slope field for

$$\frac{dy}{dx} = y$$

Use this slope field to sketch the graph of the sol'n to

(2) IVP $\begin{cases} \frac{dy}{dx} = y \\ y(0) = 1 \end{cases}$



Exercise 3 What is the formula for the function $y(x)$ solving IVP (2)?

Exercise 4 (see example 3. p.21)

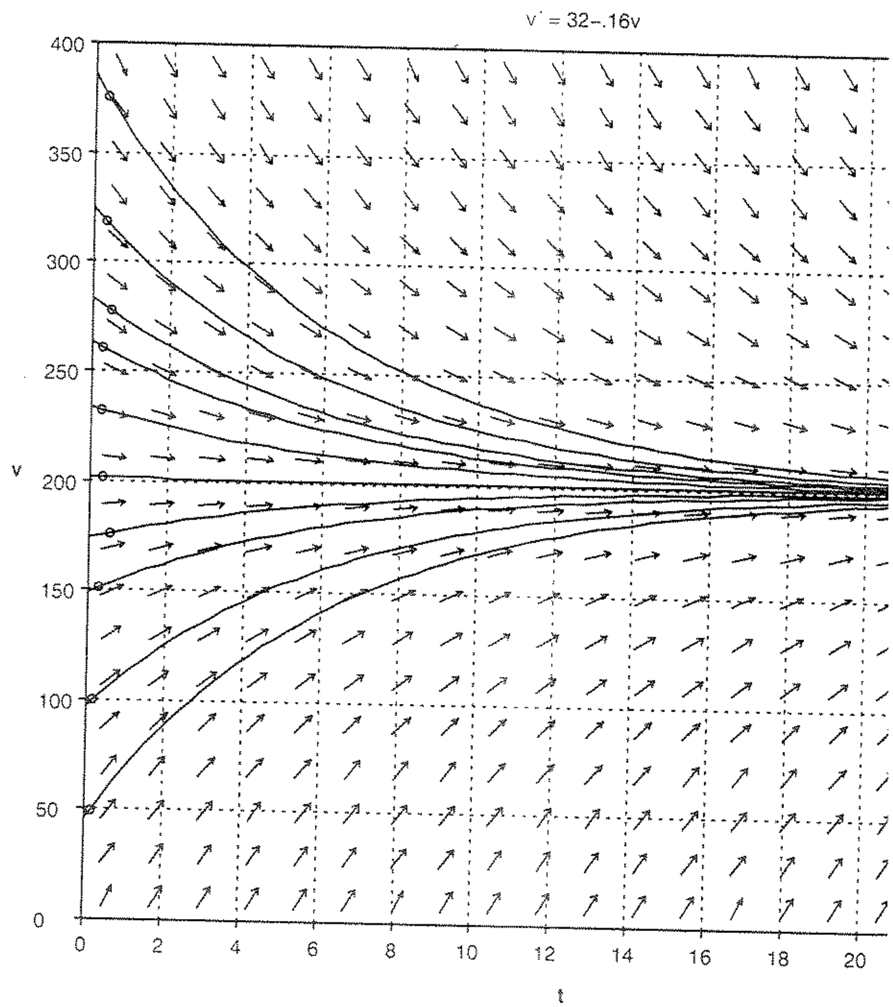
Suppose that a ball dropped or thrown from a helicopter (thrown straight down) has speed

(4) DE $\frac{dv}{dt} = 32 - .16v$

v = ft/sec
a = ft/sec²

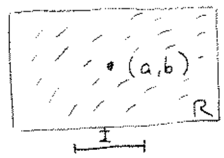
4a) discuss why DE (4) is a possible DE for the acceleration $a = \frac{dv}{dt}$.

4b) Use the slope field at the right to discuss the approximate speed of the ball when it hits ground.



Existence and uniqueness Theorem page 23
($\exists$) (!)

Suppose $f(x,y)$ and its partial derivative $D_y f(x,y)$ $\left(= \frac{\partial f}{\partial y}\right)$ are continuous on some rectangle R in the x-y plane that contains the point (a,b) in its interior



Then, for some open interval I containg a, the initial value problem

$$\begin{cases} \frac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$$

has one, and exactly one solution defined on I

Exercise 5 Discuss $\exists!$ theorem, for the IVP's we've already thought about:

(1) IVP $\begin{cases} \frac{dy}{dx} = x-3 \\ y(1) = 2 \end{cases}$

(2) IVP $\begin{cases} \frac{dy}{dx} = y \\ y(0) = 1 \end{cases}$

(4) IVP $\begin{cases} \frac{dv}{dt} = 32 - .16v \\ v(0) = 123 \end{cases}$

Exercise 6 : (see example 6 page 24)

Consider the DE $x \frac{dy}{dx} = 2y$

6a) Show $y = Cx^2$ solves this DE.

6b) Consider how ∃! Theorem applies to

(6) IVP $\begin{cases} x \frac{dy}{dx} = 2y \\ y(1) = 1 \end{cases}$

6c) on how large an interval is sol'n !?