

**MATHEMATICS 2250-1**  
**Differential Equations and Linear Algebra**  
**SYLLABUS**  
Fall semester 2008

**when:** MTWF 8:35-9:25

**where:** MWF: ST 104; T: JFB 103

**instructor:** Prof. Nick Korevaar

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**office hours:** MTWF 10:45-11:30, and by appointment

**course home page:** [www.math.utah.edu/~korevaar/2250fall08](http://www.math.utah.edu/~korevaar/2250fall08)

**text:** *Differential Equations and Linear Algebra*  
by C. Henry Edwards and David E. Penney  
ISBN = 9780555014134 (custom edition)

**prerequisites:** Math 1210-1220 (or 1250-1260 or 1270-1280, i.e. single-variable calculus.) You are also expected to have learned about vectors and parametric curves (Math 2210 or Physics 2210 or 3210).

**course outline:** This course is an introduction to differential equations, and how they are used to model problems arising in engineering and science. Linear algebra is introduced as a tool for analyzing systems of differential equations, as well as standard linear equations. Computer projects will be assigned to enhance the material.

We will cover most of chapters 1-10 in the Edwards-Penney text, in addition to two supplementary sections which cover RLC electrical circuits and additional Laplace transform material.

The course begins with first order differential equations, a subject which you touched on in Calculus. Recall that a differential equation is an equation involving an unknown function and its derivatives, that such equations often arise in science, and that the order of a differential equation is defined to be the highest order derivative occurring in the equation. The goal is to understand the solution functions to differential equations since these functions will be describing the scientific phenomena which led to the differential equation in the first place. In chapters one and two of Edwards-Penney we learn analytic techniques for solving certain first order DE's, the geometric meaning of graphs of solutions, and the numerical techniques for approximating solutions which are motivated by this geometric interpretation. We will carefully study the logistic population growth model from mathematical biology and various velocity-acceleration models from physics.

At this point in the course we will take a four week digression from differential equations, to learn the fundamentals of linear algebra in chapters 3-4. You need a basic understanding of this field of mathematics in order to talk meaningfully about the theory of higher order linear DE's and of systems of linear DE's. Chapter 3 starts out with matrix equations and the Gauss-Jordan method of solution. When you see such equations in high-school algebra you might be thinking of intersecting lines in the plane, or intersecting planes in space, or ways to balance chemical reactions, but the need to understand generally how to solve such equations is pervasive in science. From this concrete beginning we study abstract vector spaces in chapter 4. This abstract theory

is useful not only to the understanding of Euclidean space  $\mathbf{R}^n$ , but it is also the framework which allows us to understand solution spaces to systems of linear differential equations.

In Chapter 5 we will study the theory of second order and higher-order linear DE's, and focus on the ones which describe basic mechanical vibrations and electrical circuits. You may have been introduced to these equations in Calculus, and we will treat them more completely now, including a careful study of forced oscillations and resonance.

After Chapter 5 we will jump ahead to Chapter 10, the Laplace transform. This “magic” transform converts linear differential into algebraic equations, which you can solve before “inverse-transforming”. You will have to see it in action to appreciate it. You will see, for example, that this method gives a powerful way to study forced oscillations in the physically important cases that the forcing terms are square waves or other discontinuous functions. Electrical engineers often use Laplace transform techniques to understand circuits.

To successfully model more complicated physical systems like shaking in multi-story buildings, one needs the framework of linear systems of differential equations. The linear algebra of eigenvalues and eigenvectors is developed in chapter 6 as a tool for studying systems of DE's in chapter 7, where for example eigenvalues and eigenvectors will be related to fundamental modes for mass-spring systems. We will also understand these systems of DEs from the point of view of Laplace transforms.

Chapter 8, about matrix exponentials, shows the matrix algebra leads to beautiful formulas for solutions to constant coefficient linear systems of differential equations.

Our final course topic is Chapter 9, nonlinear systems and phenomena. This topic ties together many different ideas, as we discuss the stability of equilibria, based on linearization, and consider the geometry of solution trajectories in the phase portrait of autonomous systems. Many of the most interesting (complicated) dynamical systems on earth are non-linear; perhaps the most famous current example of this is global climate change. (We won't be able to study that topic in this course.)

**homework:** when assigned from the text, homework will be collected each week on Fridays, and a large proportion of the problems will be graded. You will know the assignment due on Friday by the previous Friday. You are encouraged to make friends and study groups for discussing homework, although you will each hand in your own papers (and copying someone else's work won't be productive).

The Thursday problem sessions mentioned at the beginning of this syllabus should be helpful for many of you.

whereas the Math Department Lab is located down the hall from that room, in the Rushing Student Center, between JWB and LCB. Initial tutorial times will be announced.

More information about the lab can be found at the Math Department web page for undergraduates, [www.math.utah.edu/ugrad](http://www.math.utah.edu/ugrad).

The Math tutoring center is in the Rushing Student Center, in the basement between LCB and JWB on President's Circle. You will be able to find tutors there who can help with Math 2250 homework (8 a.m.- 8 p.m. Monday-Thursday and 8 a.m.- 4 p.m. on Fridays). The page [www.math.utah.edu/ugrad/mathcenter.html](http://www.math.utah.edu/ugrad/mathcenter.html) has more information.

**computer projects:** There will be several computer projects assigned during the semester, related to the classroom material. They will be written in the software package MAPLE. In addition, you will be asked to use this computer software to check various homework calculations from throughout the course.

There is a Math Department Computer Lab at which you all automatically have accounts, and there are other labs around campus where Maple is also available, for example at the College of

Engineering and Marriott Library.

There will be tutoring center support for these projects (and for your other homework) as well. There will be introductory sessions to the Math lab and to Maple. These will be held in LCB 115, and times will be announced during the first week of classes.

**exams:** There will be two in-class midterms (closed book, scientific calculator only), as well as a final exam. The dates are as follows:

**exam 1:** Monday October 6. Probable course material is chapters 1-4.

**exam 2:** Monday November 17. Probable course material is chapters 5,10,6.

**Final Exam:** 8-10 a.m. Tuesday December 16 in our classroom ST 104 . The exam will cover the entire course, although the material after the second midterm (Chapters 7-9) can be expected to have extra weight. This is the University-scheduled time.

**grading:** Each midterm will count for 20% of your grade, the book homework and the projects will count for a total of 30%, and the final exam will make up the remaining 30% of your grade. The value of carefully working the homework problems and projects is that mathematics (like anything) must be practiced and experienced to really be learned.

University dates to keep in mind: Wednesday September 3 is the last day to drop this class, Monday September 8 is the last day to add it. Friday October 24 is the last day to withdraw. (All of these dates are easy to find from the University home page.)

**ADA statement:** The American with Disabilities Act requires that reasonable accommodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.

## Tentative Daily Schedule

exam dates fixed,  
daily subject matter approximated

M	25 Aug	1.1	differential equations and mathematical models
T	26 Aug	1.2	integral as general and particular solutions
W	27 Aug	1.3	slope fields and solution curves
F	29 Aug	1.4	separable differential equations
M	1 Sept	none	Labor Day
T	2 Sept	1.5	linear first order differential equations
W	3 Sept	2.1	population models
F	5 Sept	2.2	equilibrium solutions and stability
M	8 Sept	2.3	acceleration-velocity models
T	9 Sept	2.4-2.6	numerical solution approximations
W	10 Sept	3.1	introduction to linear systems
F	12 Sept	3.2	matrices and gaussian elimination
M	15 Sept	3.3	reduced row echelon form
T	16 Sept	3.4	matrix operations
W	17 Sept	3.5	matrix inverses
F	19 Sept	3.6	determinants
M	22 Sept	3.6	continued
T	23 Sept	4.1	3-space as a vector space
W	24 Sept	4.2	n-space and subspaces
F	26 Sept	4.3	linear combinations, dependence and independence
M	29 Sept	4.3-4.4	and bases and dimension
T	30 Sept	4.4	continued
W	1 Oct	4.5	row and column spaces
F	3 Oct	4.7	general vector spaces
M	6 Oct	Exam 1	chapters 1-4
T	7 Oct	5.1	intro to second order linear differential equations
W	8 Oct	5.2	general solutions to linear DEs
F	10 Oct	5.3	homogeneous equations with constant coefficients
M	13 Oct	none	fall break!
T	14 Oct	none	fall break!
W	15 Oct	none	fall break!
F	17 Oct	none	fall break!
M	20 Oct	5.3-5.4	mechanical vibrations
T	21 Oct	5.4	continued
W	22 Oct	5.5	nonhomogeneous equations and undetermined coefficients
F	24 Oct	5.6	forced oscillations and resonance

M	27 Oct	5.6	continued
T	28 Oct	EP 3.7	RLC circuits
W	29 Oct	EP 3.7	continued
F	31 Oct	10.1	Laplace transforms
M	3 Nov	10.2	transforms if initial value problems
T	4 Nov	10.3	translations and partial fractions
W	5 Nov	10.4	derivatives, integrals and products
F	7 Nov	10.5	periodic and piecewise continuous input functions
M	10 Nov	EP 7.6	impulses and delta functions
T	11 Nov	6.1	eigenvalues and eigenvectors
W	12 Nov	6.2	diagonalization of matrices
F	13 Nov	6.3	discrete dynamical systems
M	17 Nov	Exam 2	5, 10, 6.1-6.2
T	18 Nov	7.1	first order systems of DEs
W	19 Nov	7.2	matrices and linear systems
F	21 Nov	7.3	eigenvalue-eigenvector method for linear systems of DEs
M	24 Nov	7.4	linear second order systems of DEs
T	25 Nov	7.5	multiple eigenvalue solutions
W	26 Nov	8.1	matrix exponentials
F	28 Nov	none	Thanksgiving
M	1 Dec	8.1-8.2	and nonhomogeneous linear systems
T	2 Dec	8.2	continued
W	3 Dec	9.1	stability and the phase plane
F	5 Dec	9.2	linear and almost linear systems
M	8 dec	9.2-9.3	and ecological models
T	9 Dec	9.3-9.4	and nonlinear mechanical systems
W	10 Dec	9.4	continued
F	12 Dec	review	review
T	16 Dec	FINAL EXAM	entire course, 8-10 a.m., in our classroom