

Math 2250
Monday 9/20

Linear system of m eqns in
 n unknowns:

$$\left. \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{array} \right\} \text{LS}$$

on HW,
please work problems
in 3.1 & 3.2 by hand,
to practice Gaussian elimination.
But I encourage you to check
your answers with technology.

Goal: find all (unknown) solution vectors $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ (for which all m eqns hold
simultaneously)

the collection of these solutions is called
the solution set

The rectangle of coefficients

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

is called the coefficient matrix

If we adjoin the right-hand side vector:

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & & a_{mn} & b_m \end{array} \right]$$

this is the augmented matrix

We may do the following
equation operations without changing
the solution set:

- ① Interchange 2 eqns
- ② multiply an eqn by a non-zero constant
- ③ replace an eqn by its sum with
any multiple of another eqn

[we can combine several of these into
a single step, e.g. replace an eqn with
the sum of a non-zero mult of it, and
a multiple of another eqn]

So when we work synthetically
with the augmented matrix
we may do the corresponding
3 elementary row operations
without changing sol'n set:

- ① interchange 2 rows ("swap")
- ② multiply a row by a non-zero
constant
- ③ replace a row by its sum
with a multiple of another
row.

- Finish talking about examples on page 5 Friday notes

Example 5 p.160, illustrating Gaussian elimination for a big system)

$$\begin{aligned} x_1 - 2x_2 + 3x_3 + 2x_4 + x_5 &= 10 \\ 2x_1 - 4x_2 + 8x_3 + 3x_4 + 10x_5 &= 7 \\ 3x_1 - 6x_2 + 10x_3 + 6x_4 + 5x_5 &= 27 \end{aligned}$$

$$\begin{array}{l} \begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 2 & -4 & 8 & 3 & 10 & 7 \\ 3 & -6 & 10 & 6 & 5 & 27 \end{array} \\ \hline \begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array} \begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 2 & -1 & 8 & -13 \\ 0 & 0 & 1 & 0 & 2 & -3 \end{array} \\ \hline \begin{array}{l} R_2 \\ R_2 \end{array} \begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 2 & -1 & 8 & -13 \end{array} \\ \hline \begin{array}{l} -2R_2 + R_3 \end{array} \begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & -1 & 4 & -7 \end{array} \\ \hline \begin{array}{l} -R_3 \end{array} \begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{array} \end{array}$$

at this point we can backsolve to find sol'n:

$$\begin{aligned} x_5 &= t \\ x_4 &= 7 + 4t \\ x_3 &= -3 - 2t \\ x_2 &= s \\ x_1 &= 10 - t - 2(7 + 4t) - 3(-3 - 2t) + 2s \\ &= 5 + t(-3) + s(2) \end{aligned}$$

$$\text{so } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 7 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ -2 \\ 4 \\ 1 \end{bmatrix}, \quad s, t \in \mathbb{R}$$

row echelon form

- ① all "zero rows" (having all 0's) lie beneath all "non-zero rows"
- ② the leading non-zero entry in each non-zero row lies strictly to the right of the leading non-zero entry in the previous row.

or proceed to reduced row-echelon form, by clearing from the bottom right, moving up & to the left

$$\begin{array}{l} \begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{array} \\ \hline \begin{array}{l} 2R_3 + R_1 \\ 3R_3 + R_1 \end{array} \begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 3 & 8 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{array} \end{array}$$

reduced row echelon form ①, ② and

- ③ each leading non-zero entry is a 1 (called a "leading 1")
- ④ Every column with a leading 1 has zeroes everywhere else, i.e. above as well as below

now, backsolving is easier!

$$\begin{aligned} x_5 &= t \\ x_4 &= 7 + 4t \\ x_3 &= -3 - 2t \\ x_2 &= s \\ x_1 &= 8 - 3t + 2s \end{aligned}$$

same ans!

With Gaussian elimination we can reduce any matrix to its r.r.e.f.

Math 2250-3
reduced row echelon form
9/20/04

```
> with(linalg): #load linear algebra package
> A:=matrix(3,6,[1,-2,3,2,1,10,2,-4,8,3,10,7,
3,-6,10,6,5,27]); #3 rows, 6 columns
#is the dimension, then enter successive entries
#row by row, from left to right
```

$$A := \begin{bmatrix} 1 & -2 & 3 & 2 & 1 & 10 \\ 2 & -4 & 8 & 3 & 10 & 7 \\ 3 & -6 & 10 & 6 & 5 & 27 \end{bmatrix}$$

```
> rref(A); #get the reduced row echelon form
```

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 3 & 5 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{bmatrix}$$

You could check any one of your hw exercises this way, very quickly. For example, your last exercise this week is 3.2 #17:

```
> A:=matrix(3,5,[1,-4,-3,-3,4,2,-6,-5,-5,5,
3,-1,-4,-5,-7]);
```

$$A := \begin{bmatrix} 1 & -4 & -3 & -3 & 4 \\ 2 & -6 & -5 & -5 & 5 \\ 3 & -1 & -4 & -5 & -7 \end{bmatrix}$$

```
> rref(A);
```

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 1 & 3 & 5 \end{bmatrix}$$

```
>
```