Practice Exam #2
Math 2250-3
November 10, 2004

Exam will be closed-book and closed note. Only scientific calculators will be allowed. There will 100 points possible, with point values indicated in the right margin. The problems below are intended to be "typical", however they are not intended to be exhaustive in their scope.....please consult the review sheet for the topics you are responsible for.

1) Consider the homogeneous differential equation

$$deqtn := \left(\frac{d^2}{dt^2}\,\mathrm{x}(t)\right) + 8\left(\frac{d}{dt}\,\mathrm{x}(t)\right) + 20\,\mathrm{x}(t) = 0$$

1a) If this was modeling a mass-spring configuration like we studied in Chapter 5 of Edwards-Penney, and if the mass was 3 kg, what values of coefficient of friction and spring constant would lead to the differential equation above? (1 point for getting the units correct, 2 points for the correct numerical values).

(6 points)

1b) What kind of damping is exhibited by this mass-spring system?

(4 points)

1c) Find the general solution to this homogeneous differential equation

(5 points)

1d) Consider the same spring system, but now with a driving force F0(t)=9*cos(2t). Find the general solution to this inhomogeneous differential equation. Use the method of undetermined coefficients. Identify the steady periodic and transient pieces of the solution. Find the amplitude and phase of the steady periodic solution.

(20 points)

2) Here is a matrix:

$$A := \begin{bmatrix} 1 & 3 & -4 & -8 & 6 \\ 1 & 0 & 2 & 1 & 3 \\ 2 & 7 & -10 & -19 & 13 \end{bmatrix}$$

Here is its reduced row echelon form:

```
> rref(A);
```

$$\begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -2 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2a) Find a basis for the solution space (of homogeneous solutions) to Ax=0.

(10 points)

2b) Explain what it means for a collection of vectors to be linearly dependent or linearly independent.

(5 points)

2c) Are the first three columns of A linearly independent or linearly dependent? If they are dependent, exhibit a dependency. If they are independent, explain why.

(10 points)

2d) Explain what it means for a collection of vectors to span a vector space.

(5 points)

2e) Do the first three columns of A span all of R^3. Explain your answer.

(5 points)

2f) Find a basis for the rowspace of A.

(5 points)

3) Consider the differential equation

$$deqtn := \left(\frac{d^3}{dx^3}\,\mathrm{y}(x)\right) + 25\left(\frac{d}{dx}\,\mathrm{y}(x)\right) = 10$$

Find the solution to the initial value problem for this differential equation, with y(0)=4, D(y)(0)=0, D(D(y)(0))=10.

(25 points)