Math 2250-3
Monday Nov 29

- Finish Forced oscillations (page 3 last Wednesday)
- Ziggy experiment
- Comments on earthquake project.

We begin chapter 10 (Laplace Transforms) on Wednesday.

Calculations for Ziggy's mass-spring system

Math 2250-3,
Nov 29, 2004

The two mass, three spring system.
Data: Each ball mass is 23 grams. Each spring mass is 9 grams. (Remember, and this is a defect, our model assumes massless springs.) The springs are identical, and a mass of 50 grams stretches the spring 15.0 centimeters. (I checked this estimate before class.)

Estimate k (from the data above):

```
> k:=solve(k*.15=.05*9.81,k);
```

$$k := 3.2700$$

Time the two natural periods (which we discuss below):
(For the fast one, in my office, I got 100 cycles in about 33 sseconds. For the slow one I got 100 cycles in about 61 seconds. What do we get in class?)

Here's the model:

```
> with(linalg):
> A:=matrix(2,2,[-2*k/m, k/m,k/m,-2*k/m]);
    #this should be the "A" matrix you get for
    #our two-mass, three-spring system.
```

$$A := \begin{bmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{bmatrix}$$

```
> eigenvects(A);
```

$$\left[-\frac{k}{m}, 1, \{[1, 1]\}\right], \left[-3\frac{k}{m}, 1, \{[-1, 1]\}\right]$$

```
>
```

Predict the two natural periods from the model:

ANSWER: If you do the model correctly you will come up with natural periods of .304 and .527 seconds. I predict that these periods are too small compared to what actually happened in our experiment. What happened?

EXPLANATION: The springs actually have mass, equal to 9 grams each. This is on the same order of magnitude as the ball masses, and causes the actual experiment to run more slowly than our model predicts. In order to be more accurate the total energy of our model must account for the kinetic energy of the springs. You actually have the tools to model this more-complicated situation, using the ideas of total energy discussed in section 5.6, and a "little" Calculus. You can carry out this analysis, assuming that the spring velocity at a point on the spring linearly interpolates the velocity of the wall and mass (or mass and mass) which bounds it. It turns out that this gives the same eigenvectors, but different eigenvalues, namely

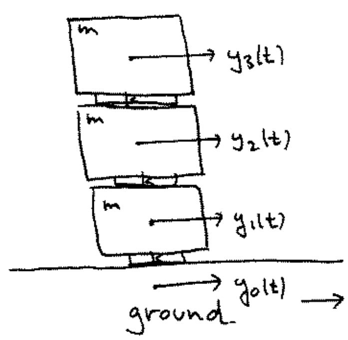
$$\lambda_1 = -\frac{k}{m + \frac{5}{6} m_s}$$

$$\lambda_2 = -3\,\frac{k}{m + \frac{1}{2} m_s}$$

If you use these values, then you get period predictions of .61 and .33 seconds per cycle. Is that closer?

Earthquake Project Comments

consider a 3 story building (in project is 7 stories) modeled as a mass-spring model, with each story having mass m, and floor junctions modeled as springs with constants k :



$\rightarrow$ transverse (shaking) wave

$E\cos\omega t$

$y_0(t) = E\cos\omega t$ so

$$y_0'' = -E\omega^2\cos\omega t$$
$$m y_1'' = -k(y_1-y_0) + k(y_1-y_2)$$
$$m y_2'' = -k(y_2-y_1) - k(y_2-y_3)$$
$$m y_3'' = -k(y_3-y_2)$$

reduction of fcns

let

$$x_0 = y_0 - E\cos\omega t \equiv 0$$
$$x_1 = y_1 - E\cos\omega t$$
$$x_2 = y_2 - E\cos\omega t$$
$$x_3 = y_3 - E\cos\omega t$$

$$x_0 \equiv 0$$
$$m x_1'' = m y_1'' + mE\omega^2\cos\omega t$$
$$m x_2'' = m y_2'' + mE\omega^2\cos\omega t$$
$$m x_3'' = m y_3'' + mE\omega^2\cos\omega t$$

so

$$m x_1'' = -k(x_1-0) - k(x_1-x_2) + mE\omega^2\cos\omega t$$
$$m x_2'' = -k(x_2-x_1) - k(x_2-x_3) + mE\omega^2\cos\omega t$$
$$m x_3'' = -k(x_3-x_2) + mE\omega^2\cos\omega t$$

so the x's are the displacements from equilibrium as measured by an observer on the ground

Notice $y_i - y_j = x_i - x_j$

$$\boxed{\begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} = \begin{bmatrix} -2k/m & k/m & 0 \\ k/m & -2k/m & k/m \\ 0 & k/m & -k/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + E\omega^2\cos\omega t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

it's as if all floors are being forced – book calls this inertial forcing due to frame of reference