

MATHEMATICS 2250-3
Ordinary Differential Equations and Linear Algebra
Fall semester 2004

text: *Differential Equations & Linear Algebra, second edition*
by C.H. Edwards Jr. and David E. Penney
when: MWF 12:55-1:45
where: EMCB 105
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office hours: M, W, F 10:35-10:55 a.m., T 11-11:50 a.m., 1:00-1:50 p.m.
and by appointment.
2250-3 home page: www.math.utah.edu/~korevaar/2250fall04

prerequisites: Math 1210 and 1220 or equivalent (first-year Calculus, with a very brief introduction to linear differential equations). In addition you are expected to be familiar with vectors, curves, velocity (tangent), and acceleration vectors, either from Physics 2210 or Math 2210, or their equivalents (e.g. Math 1250-1260 or 1270-1280).

course outline: This course is an introduction to differential equations, and how they are used to model problems arising in engineering and science. Linear algebra is introduced as a tool for analyzing systems of differential equations, as well as standard linear equations. Computer projects will be assigned to enhance the material.

We will cover most of chapters 1-7 and 10 in the Edwards-Penney text. The course begins with first order differential equations, a subject which you touched on in Calculus. Recall that a differential equation is an equation involving an unknown function and its derivatives, that such equations often arise in science, and that the order of a differential equation is defined to be the highest order derivative occurring in the equation. The goal is to understand the solution functions to differential equations since these functions will be describing the scientific phenomena which led to the differential equation in the first place. In chapters one and two of Edwards-Penney we learn analytic techniques for solving certain first order DE's, the geometric meaning of graphs of solutions, and the numerical techniques for approximating solutions which are motivated by this geometric interpretation. We will carefully study the logistic population growth model from mathematical biology and various velocity-acceleration models from physics.

At this point in the course we will take a four week digression from differential equations, to learn the fundamentals of linear algebra in chapters 3-4. You need a basic understanding of this field of mathematics in order to talk meaningfully about the theory of higher order linear DE's and of systems of linear DE's. Chapter 3 starts out with matrix equations and the Gauss-Jordan method of solution. When you see such equations in high-school algebra you might be thinking of intersecting lines in the plane, or intersecting planes in space, or ways to balance chemical reactions, but the need to understand generally how to solve such equations is pervasive in science. From this concrete beginning we study abstract vector spaces in chapter 4. This abstract theory is useful not only to the understanding of Euclidean space \mathbf{R}^n , but it is also the framework which allows us to understand solution spaces to systems of linear differential equations.

In chapter 5 we will study the theory of second order and higher-order linear DE's, and focus on the ones which describe basic mechanical vibrations. You were introduced to these equations in Calculus, and we will treat them more completely now, including a careful study of forced oscillations and resonance.

To successfully model more complicated physical systems like shaking in multi-story buildings, one needs the framework of linear systems of differential equations. The linear algebra of eigenvalues and eigenvectors is developed in chapter 6 as a tool for studying systems of DE's in chapter 7, where for example eigenvalues and eigenvectors will be related to fundamental modes for mass-spring systems.

Our final course topic is the Laplace transform and its applications to the study of linear DE's, in chapter 10. This "magic" transform takes differential equations and "transforms" them into algebraic equations. You

will have to see it in action to appreciate it. You will see, for example, that this method gives a powerful way to study forced oscillations in the physically important cases that the forcing terms are square waves or other discontinuous functions. Electrical engineers often use Laplace transform techniques to understand circuits.

computer projects: There will be four computer projects assigned during the semester, related to the classroom material. They will be written in the software package MAPLE. In addition, you will be asked to use this computer software to check various homework calculations from throughout the course. There is a Math Department Computer Lab at which you all automatically have accounts, and there are other labs around campus where Maple is also available, for example at the College of Engineering and Marriott Library. There will tutoring center support for these projects (and for your other homework) as well. There will be introductory sessions to the lab and to Maple. These will be held in LCB 115, whereas the Math Department Lab is located down the hall in the Rushing Student Center, between JWB and LCB. Initial tutorial times are Tuesday August 31, 11:50-12:40; Wednesday September 1, 2-2:50; Friday September 3, 10:45-11:35 and 11:50-12:40. More tutorials will be added as necessary. More information about the lab can be found at the Math Department web page for undergraduates, www.math.utah.edu/ugrad .

tutoring/help sessions: There will be special help sessions for Math 2250 students, and these will be held on Tuesdays. (Homework is due on Wednesdays.) The sessions are scheduled as follows: In EMCB 101 from 7:30-8:20 a.m.; in LS 101 from 10:45-11:35 a.m.; in EMCB 105 from 12:55-1:45 p.m.; in LCB 219 from 5-5:50 p.m. Students may attend as many or as few of these sessions as they wish. If 5 or more students get together and make a request to me or another instructor, we can schedule additional sessions.

In addition to the special 2250 sessions, the Math Department Tutoring Center is located in Rushing Student Center and is open for free tutoring from 8 a.m. to 8 p.m. on M-Th, and from 8 a.m. to 6 p.m. on Friday. Some, but not all of the math tutors welcome questions from Math 2250 students. To see the times and specialities of various tutors, consult the web address www.math.utah.edu/ugrad/tutoring.html .

grading: There will be two midterms, a comprehensive final examination, and homework. Each midterm will count for 20% of your grade, homework (from the book and on the computer) will count for 30%, and the final exam will make up the remaining 30%. The book homework will be assigned daily and collected weekly, on Wednesdays. The four Maple project assignment and collection dates are shown on the syllabus. A homework grader will partially grade your book homework and Maple assignments. The value of carefully working homework problems is that mathematics (like anything) must be practiced and experienced to be learned.

It is the Math Department policy, and mine as well, to grant any withdrawl request until the University deadline of Friday October 22.

ADA statement: The American with Disabilities Act requires that reasonable accomodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.

Tentative Daily Schedule

exam dates fixed, Maple start dates fixed
daily subject matter approximated

W	25 Aug	1.1	modeling and differential equations
F	27 Aug	1.2	integral solutions
M	30 Aug	1.3	direction fields and solution curves
W	1 Sept	1.4	separable equations
		Maple I	<i>begin</i> project on chapter 1-2 material
F	3 Sept	1.5	first order linear DE's
M	6 Sept	none	Labor Day
W	8 Sept	2.1	population models
F	10 Sept	2.2	equilibrium solutions and stability
M	13 Sept	2.3	acceleration-velocity models
		Maple Ia due	<i>hand in</i> first part of Maple project
W	15 Sept	2.4 & 2.6	numerical techniques: Euler and Runge Kutta
F	17 Sept	3.1	introduction to linear systems
M	20 Sept	3.2	matrices and Gaussian elimination
		Maple Ib due	<i>hand in</i> second part of Maple project
W	22 Sept	3.3	reduced row-echelon form
F	24 Sept	3.4	matrix operations
M	27 Sept	3.5	inverses of matrices
		Maple II	<i>begin</i> project on linear algebra
W	29 Sept	3.6	determinants
F	1 Oct	4.1	the vector space \mathbf{R}^3
M	4 Oct	exam 1	1-3
W	6 Oct	4.2	the vector space \mathbf{R}^n and subspaces
F	8 Oct	none	fall break day
M	11 Oct	4.3	linear combinations and linear independence
W	13 Oct	4.4	bases and dimension of vector spaces
F	15 Oct	4.5	row and column spaces
M	18 Oct	4.6	orthogonality
W	20 Oct	4.7	general vector spaces
F	22 Oct	5.1	second-order linear differential equations
M	25 Oct	5.2	general solutions of linear equations
W	27 Oct	5.3	homogeneous equations with constant coefficients
		Maple II due	<i>hand in</i> Maple II
		Maple III	<i>begin</i> mechanical oscillations project
F	29 Oct	5.4	mechanical vibrations

M	1 Nov	5.5	undetermined coefficients and variation of parameters
W	3 Nov	5.5-5.6	and forced oscillations and resonance
F	5 Nov	5.6	continued
M	8 Nov	6.1	introduction to eigenvalues
W	10 Nov	7.1	first order systems and applications
		Maple III due	<i>hand in</i> third Maple project
F	12 Nov	exam 2	4-6
M	15 Nov	7.2	matrices and linear systems
		Maple IV	<i>begin</i> earthquake project
W	17 Nov	7.3	eigenvalue method for linear systems
F	19 Nov	7.3-7.4	& second order systems and mechanical applications
M	22 Nov	7.4	continued
W	24 Nov	10.1	Laplace transforms and inverse transforms
F	26 Nov	none	Thanksgiving recess
M	29 Nov	10.2	transformation of initial value problems
W	1 Dec	10.3	translation and partial fractions
F	3 Dec	10.4	derivatives, integrals, products of transforms
M	6 Dec	10.4-10.5	& periodic and piecewise continuous forcing functions
		Maple IV due	<i>hand in</i> fourth Maple project
W	8 Dec	10.5	continued
F	10 Dec	none	University reading day
Tues	14 Dec	FINAL EXAM	entire course 1-3 p.m.