Math 2250-3
September 17, 2003
Maple linear algebra

```
> with(linalg): #load linear algebra package
```

Problem #17, section 3.2, page 160. This was a homework problem.

```
> A:= matrix(3,4,[1,-4,-3,-3,2,-6,-5,-5,3,-1,-4,-5]);
      #the coefficient matrix
```

$$A := \begin{bmatrix} 1 & -4 & -3 & -3 \\ 2 & -6 & -5 & -5 \\ 3 & -1 & -4 & -5 \end{bmatrix}$$

```
> b:=vector([2,5,-7]);  #the right hand side
```

$$b := [2, 5, -7]$$

```
> Aaugb:=augment(A,b);  #the augmented matrix...
      #you could have entered it directly too.
```

$$Aaugb := \begin{bmatrix} 1 & -4 & -3 & -3 & 2 \\ 2 & -6 & -5 & -5 & 5 \\ 3 & -1 & -4 & -5 & -7 \end{bmatrix}$$

```
> rref(Aaugb);  #compute the reduced row echelon form
```

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 41 \\ 0 & 1 & 0 & -1 & -18 \\ 0 & 0 & 1 & 3 & 37 \end{bmatrix}$$

From rref(Aaugb) you can read off the solution to the original problem. Each column (from A) without a leading 1 gives you a free parameter in the solution. so we see that x4=t, x3=37-3t, x2=-18+t, x1=41-2t is the general solution. We could also write this in vector form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 41 \\ -18 \\ 37 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ -3 \\ 1 \end{bmatrix}$$

Important conceptual question:

Linear systems in which right hand side vector equals zero are called homogeneous linear systems. Otherwise they are called inhomogeneous. Notice that in the problem above, the constant vector [41,-18,37,0] is a particular solution to the inhomogeneous system. Notice that the set of multiples {t*[-2,1,-3,1]} is the general solution to the homogeneous problem. (Why?) So the general solution to our system was a particular solution to it plus the general solution to the homogeneous equation. Was this an accident? We'll come back to this.

By the way, Maple will go ahead and solve the linear system directly if you ask it to:

```
> linsolve(A,b);
```

$$[41 - 2\ \_t_1,\ -18 + \_t_1,\ 37 - 3\ \_t_1,\ \_t_1]$$

Would this have been the same answer as:

$$[5 - 2\ t,\ t,\ -17 - 3\ t,\ 18 + t]$$

Check!!!!

Using rrref(A) to discern general facts about the solutions to Ax=b:

The reduced row echelon form of a matrix tells you a lot about possible solutions to the matrix equation Ax=b. What can you say in the following situations. Notice, I HAVE NOT augmented the matrix before finding the reduced row echelon form:

```
> A:=matrix(3,5,[2,7,-10,-19,13,1,3,-4,-8,6,1,0,2,1,3]);
```

$$A := \begin{bmatrix} 2 & 7 & -10 & -19 & 13 \\ 1 & 3 & -4 & -8 & 6 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix}$$

```
> RREFA:=rref(A); #the actual command is lower case!
```

$$RREFA := \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -2 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

How many solutions to the homogeneous problem Ax=0? How many free parameters in the solution?

Is the inhomogeneous problem Ax=b always solvable?

When it is solvable, how many solutions are there?

```
> B:=matrix(3,2,[1,2,-1,3,4,2]);
```

$$B := \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 4 & 2 \end{bmatrix}$$

```
> RREFB:=rref(B);
```

$$RREFB := \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

How many solutions to the homogeneous problem Bx=0?

Is the inhomogeneous problem Bx=b always solvable?

When it is solvable, how many solutions does it have?

```
> C:=matrix(4,4,[1,0,-1,1,22,-1,3,5,7,4,6,2,3,5,7,13]);
```

$$C := \begin{bmatrix} 1 & 0 & -1 & 1 \\ 22 & -1 & 3 & 5 \\ 7 & 4 & 6 & 2 \\ 3 & 5 & 7 & 13 \end{bmatrix}$$

```
> RREFC:=rref(C);
```

$$RREFC := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Square matrices with 1's down the diagonal (which runs from the upper left to lower right corner) are special. They are called identity matrices.

How many solutions to the homogeneous problem Cx=0?

Is the inhomogeneous problem Cx=b always solvable?

How many solutions?

What are your general conclusions?

(1) What conditions on rref(A) guarantee that the homogeneous equation Ax=0 has infinitely many solutions?

(2) What conditions on the dimensions of A force infinitely many solutions to the homogeneous problem regardless of the individual entries of A?

(3) What conditions on rref(A) guarantee that solutions x to Ax=b are always unique (if they exist)?

(4) If A is a square matrix, what can you say about solutions to Ax=b when
(4a) rref(A) is the identity matrix

(4b) rref(A) is not the identity matrix?