Math 2250

Earthquake project

November 2002

Name ________________________________ Class Time _________
Project 3. Solve problems 3.1 to 3.6. The problem headers:

______ PROBLEM 3.1. BUILDING MODEL FOR AN EARTHQUAKE.
______ PROBLEM 3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
______ PROBLEM 3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
______ PROBLEM 3.4. PRACTICAL RESONANCE.
______ PROBLEM 3.5. EARTHQUAKE DAMAGE.
______ PROBLEM 3.6. SIX FLOORS.

```
> with(linalg):
```

3.1. BUILDING MODEL FOR AN EARTHQUAKE.

Refer to the textbook of Edwards-Penney, section 7.4, page 437. Consider a building with 7 floors.

Let the mass in slugs of each story be $m = 1000.0$ and let the spring constant be $k = 10000$ lbs/foot. Define the 7 by 7 mass matrix $M$ and Hooke's matrix $K$ for this system and convert $Mx'' = Kx$ into the system $x'' = Ax$ where $A$ is defined by textbook equation (1) , page 437.

PROBLEM 3.1
Find the eigenvalues of the matrix A to six digits, using the Maple command `eigenvals(A)`. Justify in particular that all seven eigenvalues are negative by direct computation.
# Sample Maple code for a model with 4 floors.
# Use maple help to learn about evalf and eigenvals.
A:=matrix([ [-20,10,0,0], [10,-20,10,0], [0,10,-20,10], [0,0,10,-10]]);
evalf(eigenvals(A));

```
> # Problem 3.1
```

3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
Refer to figure 7.4.17, page 437.

PROBLEM 3.2.
Find the natural angular frequencies $\omega = \sqrt{-\lambda}$ for the seven story building and also the corresponding periods $2\pi/\omega$, accurate to six digits. Display the answers in a table . The answers appear in Figure 7.4.17, page 437.
# Sample code for a 4x3 table.
# Use maple help to learn about nops and printf.
ev:=[-10,-1.206147582,-35.32088886,-23.47296354]: n:=nops(ev):
Omega:=lambda -> sqrt(-lambda):
format:="%10.6f %10.6f %10.6f\n":
seq(printf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..n );

```
> # Problem 3.2
```

3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
Consider the forced equation $x' = Ax + \cos(wt)b$ where $b$ is a constant vector. The earthquake's ground vibration is accounted for by the extra term $\cos(wt)b$, which has period $T = 2\pi/w$. The solution x(t) is the 7-vector of excursions from equilibrium of the corresponding 7 floors. Sought here is not the general solution, which certainly contains transient terms, but rather the steady-state periodic solution, which is known from the theory to have the form $x(t) = \cos(wt)c$ for some vector $c$ that depends only on $A$ and $b$.

PROBLEM 3.3.
Define `b:=0.25*w*w*vector([1,1,1,1,1,1,1]):` in Maple and find the vector $c$ in the undetermined coefficients solution $x(t) = \cos(wt)c$. Vector $c$ depends on $w$. As outlined in the textbook, vector $c$ can be found by solving the linear algebra problem $-w^2c = Ac + b$; see page 433. Don't print $c$, as it is too complex; instead, print `c[1]` as an illustration.

```
# Sample code for defining b and A, then solving for c in the 4-floor case.
# See maple help to learn about vector and linsolve.
w:='w': u:=w*w: b:=0.25*u*vector([1,1,1,1]):
Au:=matrix([ [-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10], [0,0,10,-10+u]]);
c:=linsolve(Au,-b):
evalf(c[1],2);
> # PROBLEM 3.3
```

3.4 PRACTICAL RESONANCE.
Consider the forced equation $x' = Ax + \cos(wt)b$ of 3.3 above. Practical resonance can occur if a component of $x(t)$ has large amplitude compared to the vector norm of $b$. For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy the building.

PROBLEM 3.4.
Let Max(c) denote the maximum modulus of the components of vector c. Plot $g(T) = Max(c(w))$ with $w = 2\pi/T$ for periods $T = 0$ to $T = 6$, ordinates $Max = 0$ to $Max = 10$, the vector $c(w)$ being the answer produced in 3.3 above. Compare your figure to the textbook Figure 7.4.18, page 438.

```
# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, vector, subs and linsolve.
with(linalg):
w:='w': Max:= c -> norm(c,infinity); u:=w*w:
b:=0.25*w*w*vector([1,1,1,1]):
Au:=matrix([ [-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10], [0,0,10,-10+u]]);
C:=ww -> subs(w=ww,linsolve(Au,-b)):
plot(Max(C(2*Pi/r)),r=0..6,0..10,numpoints=150);
> # PROBLEM 3.4.  WARNING: Save your file often!!!
```

3.5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of 3.4 can be used to detect the likelihood of earthquake damage for a given ground vibration of period $T$. A ground vibration $(1/4)\cos(wt)$, $T = 2\pi/w$, will be assumed, as in 3.4.

PROBLEM 3.5.
(a) Replot the amplitudes in 3.4 for periods 1.14 to 4 and amplitudes 5 to 10. There will be four spikes.
(b) Create four zoom-in plots, one for each spike, choosing a $T$-interval that shows the full spike.
(c) Determine from the four zoom-in plots approximate intervals for the period $T$ such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet.

# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet, periods 1.97 to 2.01.
with(linalg): w:='w': Max:= c -> norm(c,infinity); u:=w*w:
Au:=matrix([ [-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10], [0,0,10,-10+u]]);
b:=0.25*w*w*vector([1,1,1,1]):
C:=ww -> subs(w=ww,linsolve(Au,-b)):
plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);

```
> # PROBLEM 3.5.  WARNING: Save your file often!!
> #(a) Plot four spikes on separate graphs
> #(b) Plot four zoom-in graphs.
> #(c) Print period ranges.
```

3.6. SIX FLOORS.
Consider a building with six floors each weighing 50 tons. Assume each floor corresponds to a restoring Hooke's force with constant $k = 5$ tons/foot. Assume that ground vibrations from the earthquake are modeled by $(1/4)\cos(wt)$ with period $T = 2\pi/w$ (same as the 7-floor model above).

PROBLEM 3.6.
Model the 6-floor problem in Maple. Plot the maximum amplitudes against the period 0 to 6 and amplitude 4 to 10. Determine from the graphic the period ranges which cause the amplitude plot to spike above 4 feet. Sanity check: m=3125, and the 6x6 matrix contains fraction 16/5. There are five spikes.

```
> # PROBLEM 3.6.  WARNING: Save your file often!!
> # Define k, m and the 6x6 matrix A.
> # Amplitude plot for T=0..6,C=4..10
> # Plot five zoom-in graphs
> # From the graphics, five T-ranges give amplitude # spikes above 4 feet.  These are determined
  by left # mouse-clicks on the graph, so they are approximate values only.
```