Name..
I.D. number...

Math 2250-4
Practice Exam

September 28, 2001

This is a practice exam, to give you an idea of the actual exam length and topic possibilities. Of course, the topics which will appear on actual exam will not be exactly the same as the ones which appear here!

This exam is closed-book and closed-note. You may use a scientific calculator, but not one which is capable of graphing or of solving differential or linear algebra equations. **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** There are 100 points possible, and the point values for each problem are indicated in the right-hand margin.

Good Luck!

1) Consider the differential equation

$$\frac{dP}{dt} = -P^2 + 4P - 3$$

which models a certain population problem.

1a) Find the equilibrium solutions.

(5 points)

1b) Sketch the slope field for this differential equation. Onto the slope field sketch graphs of the solutions to the three initial value problems with P(0)=0, P(0)=2, P(0)=4. (You don't need formulas for the solutions to make the sketches!)

(10 points)

1c) Which of the equilibrium solutions are stable? Which are unstable?

(5 points)

1d) Give a population model which leads to differential equations of this type. Be as precise as you can, so that you can account for the signs of all three terms on the right-hand side of the differential equation.

(5 points)

1e) Find a explicit solution to the initial value problem for this differential equation, with P(0)=2. Verify that your limiting population agrees with what your sketch predicted in part 1b).

(15 points)

2) Consider a brine tank which holds 15,000 gallons of continuously-mixed liquid. Let x(t) be the amount of salt (in pounds) in the tank at time t. The in-flow and out-flow rates are both 150 gallons/hour, and if the concentration of salt flowing in is 1 pound per 10 gallons of water.

2a) Explain how the information above leads to the differential equation

$$\frac{dx}{dt} + .01\,x = 15$$

(5 points)

2b) Solve the initial value problem for this differential equation, assuming that at time t=0 there is no salt in the water.

(10 points)

2c) What is the limiting amount of salt as t approaches infinity?

(5 points)

3) Consider the matrix

$$A := \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

3a) Compute determinant(A). What does this computation tell you about whether A is singular or nonsingular?

(5 points)

3b) Find the inverse matrix to A, using the row-operation algorithm. Remember to show all work, as always. Hint: The correct inverse matrix has no fractions in it, which you could deduce from your answer to part (4a).

(15 points)

3c) Recompute the inverse, using the adjoint formula. Show the cofactor matrix as well as the adjoint matrix as intermediate steps, so that I can check your work.

(15 points)

3d) Use the inverse matrix from (3b) or (3c) to solve the system

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

(5 points)