

Math 2250 Fall 2001
Project I
SolutionTemplate

1) Type in your name and student number here:

You are to do the section 1.5 computer project, using this template. When you are done you should print out a copy to hand in to your Instructor. A preliminary discussion, in which the text discussion of pages 55-57 is expanded to include Maple commands, is found at the end of the tutorial which accompanies this project. It is assumed that you have already worked through that. Both the tutorial and this template can be downloaded by following the links from our course home page, at <http://www.math.utah.edu/~korevaar/2250fall01.html>

SCENARIO:

It is not summer in Georgia. It is winter in Salt Lake City. We will dream it is early spring, (around equinox), and that the 5-day forecast in the newspaper says the weather will be stable, with lows of 26 degrees, and highs of 60 degrees. As in the text and for the sake of simplicity, we will assume a sinusoidal daily temperature oscillation, except our low will be at 2 a.m, and our high will be at 2 p.m.

We must leave town for 3 days, and are deciding whether to turn off the heat while we are gone. The question we wonder about is, will the water pipes in the house freeze if we do turn off the heat?

2) Review the model which leads to equation (3) on page 56. Use integrals #49 and 50 and the algorithm for solving first order linear DE's on pages 44-45, to solve (3) by hand, keeping all parameters as letters. Staple your work onto the printout of your completed project which you hand in. Your answer should agree with equation (4) on page 56. You will see that the constant c_0 in that answer is the constant C of integration you obtain when you follow the solution recipe for linear DE's. You find its value in terms of the initial condition $u(0)=u_0$ by plugging in $u=u_0$, $t=0$, as usual. You will discover that the text writes c_0 incorrectly: there is a hidden minus sign which has become glued to the fraction which follows a_0 , in the book's formula, i.e. the correct formula for c_0 is

$$c_0 := \frac{k \omega b_1 - k^2 a_1}{k^2 + \omega^2} + u_0 - a_0$$

[> restart:with(DEtools):with(plots):

3) Use `dsolve` to have Maple find the solution to (3), with $u(0)=u_0$. You might have already done this in the tutorial, in which case you can copy the appropriate commands from there, paste them in here, and re-execute them. Check that your solution agrees with your hand work above, as well as the (corrected) text.

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4) Figure out the parameter values for the Salt Lake City temperature, as modeled above. Of course, ω will still be $\pi/12$, but now you have different daily average temperature, amplitude, and phase than was used in equation (2) on page 56, when summer in Georgia was being modeled. Fill the new values in below. Then work out by hand, using the cosine addition formula, the values you should take for a_0 , a_1 , b_1 , so that $A(t)$ is given by (1) on page 56, and so that the differential equation (3) also has the correct parameter values.

Average Temperature =
 Temperature variation amplitude =
 Phase delay (was 4 hours in Georgia example) =

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[ > omega:=Pi/12.0;
  a0:=
    #don't forget the semicolons
  a1:=
  b1:=
  A:=t->a0 + a1*cos(omega*t) + b1*sin(omega*t);
```

We will assume your house is moderately well-insulated, so that $k=0.3$:

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[ > k:=0.3;
```

5) Find the solution to (3) with the particular parameter choices you made above. You should get a solution function like (5) on page 56, but reflecting the Salt Lake City temperatures and the new insulation parameter.

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6) Identify the part of your solution which persists as t approaches infinity, i.e. the steady periodic solution. Your formula should have the same character as equation (6) on page 57.

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7) Write the steady periodic solution in the form of equation (7) on page 57, so that you can see the time delay for the inside temperature. You need to use the cosine addition formula. The text has a discussion on page 315 which may help you. It is shown there that if

$$x(t) := A \cos(\omega t) + B \sin(\omega t)$$

then also

$$x(t) := t \rightarrow C \cos(\omega t - \alpha)$$

where the right triangle of Figure 5.4.4 page 315 summarizes the relationships between A,B, C, and alpha.

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8) Assume that your heater shut off at midnight, with the inside temperature equal to 70 degrees. Create a plot like Figure 1.5.10 which displays the inside temperature and the outside ambient temperature for the next three days.

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9) Create a picture like Figure 1.5.9, which also includes the slope field for this differential equation, with our Salt Lake City parameters. Choose initial temperatures between 40 and 70 degrees, in 5 degree increments. Label the maximum and minimum temperature times by hand, on the display printout, before you hand in the project.

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10) So, based on your work in this project, how likely do you think it is that the pipes will freeze if the heater is turned off for 3 days starting at midnight? Explain your reasoning.