Math 2250
Earthquake project
November 2001

Enter your name here: ??????????????????????????????

Project 3. Please fill in all areas below marked ????? and solve problems 3.1 to 3.6. The problem headers:

_______ PROBLEM 3.1. BUILDING MODEL FOR AN EARTHQUAKE.
_______ PROBLEM 3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
_______ PROBLEM 3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
_______ PROBLEM 3.4. PRACTICAL RESONANCE.
_______ PROBLEM 3.5. EARTHQUAKE DAMAGE.
_______ PROBLEM 3.6. FIVE FLOORS.

```
> restart:with(plots):with(linalg):
```

3.1. BUILDING MODEL FOR AN EARTHQUAKE.
Refer to the textbook of Edwards-Penney, section 7.4, page 437. Consider a building with 7 floors.

Let the mass in slugs of each story be m=1000.0 and let the spring constant be k=10000.0 (lbs/foot). Define the 7 by 7 mass matrix M and Hooke's matrix K for this system and convert Mx''=Kx into the system x''=Ax where A is defined by textbook equation (1) , page 437.

PROBLEM 3.1
Find the eigenvalues of the matrix A to six digits, using the Maple command "eigenvals(A)." Justify in particular that all seven eigenvalues are negative by direct computation.

```
# Sample Maple code for a model with 4 floors.
# Use maple help to learn about evalf and eigenvals.
 A:=matrix([ [-20,10,0,0], [10,-20,10,0], [0,10,-20,10], [0,0,10,-10]]);
 evalf(eigenvals(A));
```

```
> # Problem 3.1
>
```

3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
Refer to figure 7.4.17, page 437.

PROBLEM 3.2.
Find the natural angular frequencies omega=sqrt(-lambda) for the seven story building and also the corresponding periods
2PI/omega, accurate to six digits. Display the answers in a table .

```
# Sample code for a 4x3 table.
# Use maple help to learn about nops, printf.
 ev:=[-38.3,-33.4,-26.2,-17.9]:
 Omega:=lambda -> sqrt(-lambda):
 format:="%10.6f  %10.6f  %10.6f\n":
 seq(printf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..nops(ev));
```

```
> # Problem 3.2
>
```

3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
Consider the forced equation x'=Ax+cos(wt)b where b is a constant vector. The earthquake's ground vibration is accounted for by the extra term cos(wt)b, which has period T=2Pi/w.
The solution x(t) is the 7-vector of excursions from equilibrium of the corresponding 7 floors.
Sought here is not the general solution which contains the general solution to
to homogeneous problem, but rather the particular periodic solution with angular freqency w,
which is known from the theory to have the form x(t)=cos(wt)c for some vector c that
depends only on A and b. The idea is that in any real problem there would be a small amount
of damping, which would lead to transient homogeneous solution, and steady periodic solution close to the one you are finding for this undamped problem.

PROBLEM 3.3.
Define b:=(1/4)*w*w*vector([1,1,1,1,1,1,1]): in Maple and find the vector c in the undetermined coefficients solution x(t)=cos(wt)c. Vector c depends on w. As outlined in the textbook, vector c can be found by solving the linear algebra problem -w^2 c = Ac + b; see page 433. Don't print c, as it is too complex; instead, print c[1] as an illustration.

```
# Sample code for defining b and A, then solving for c in the 4-floor case.
# See maple help to learn about evalm, diag, vector.
  w:='w':
  b:=0.25*w*w*vector([1,1,1,1]):
  A:=matrix([ [-20,10,0,0], [10,-20,10,0], [0,10,-20,10], [0,0,10,-10]]);
  c:=linsolve(evalm(A+w*w*diag(1,1,1,1)),-b):
  evalf(c[1],6);
```

```
> # PROBLEM 3.3
>
```

3.4 PRACTICAL RESONANCE.
Consider the forced equation x'=Ax+cos(wt)b of 3.3 above with b:=0.25*w*w*vector([1,1,1,1,1,1,1]). Practical resonance occurs when any component of x(t) has large amplitude compared to the vector norm of the forcing amplitudes in b. For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy the building.

PROBLEM 3.4.
Let Max(c) denote the maximum modulus of the components of vector c. Plot g(T)=Max(c(w)) with w=(2*Pi)/T for periods T=0 to T=6, ordinates Max=0 to Max=10, the vector c(w) being the answer produced in 3.3 above. Compare your figure to the textbook Figure 7.4.18, page 438.

```
# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, diag, vector, subs, evalm and linsolve.
  with(linalg):
  w:='w': Max:= c -> norm(c,infinity);
  B:=w*w*diag(1,1,1,1): b:=0.25*w*w*vector([1,1,1,1]):
  C:=ww -> subs(w=ww,linsolve(evalm(A+B),-b)):
  plot(Max(C(2*Pi/r)),r=0..6,0..10);
```

```
> restart:with(plots):with(linalg):
> # PROBLEM 3.4. WARNING: Save your file often!!!
>
```

3.5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of 3.4 can be used to detect the likelihood of earthquake damage for a given
ground vibration of period T. A ground vibration (1/4)cos(wt), T=2*Pi/w, will be assumed, as in 3.4.

PROBLEM 3.5.
Replot the amplitudes in 3.4 for periods 0 to 6 and amplitudes 5 to 10. There will be three spikes. Zoom-in on each spike, choosing a T-interval that shows the full spike. Determine from the three zoom-in plots approximate intervals for the period T such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet.

```
# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet, periods 1.4 to 1.5.
plot(Max(C(2*Pi/r)),r=1.4..1.5,5..10);
```

```
> # PROBLEM 3.5. WARNING: Save your file often!!
> # Plot three spikes on one graph.
> # Plot three zoom-in graphs.
> # Print period ranges.
```

3.6. FIVE FLOORS.
Consider a building with five floors each weighing 20 tons. Assume each floor corresponds to a restoring
Hooke's force with constant k=4 tons/foot. Assume that ground vibrations from the earthquake are modeled by
(1/4)cos(wt) with period T=2*Pi/w (same as the 7-floor model above).

PROBLEM 3.6.
Model the 5-floor problem in Maple. Plot the maximum amplitudes against the period 0 to 6 and amplitude
4 to 10. Determine from the graphic the period ranges which cause the amplitude plot to spike above 4 feet.

```
> restart:with(plots):with(linalg):
> # PROBLEM 3.6. WARNING: Save your file often!!
> # Define k=??? and m=???, then matrix A=???.

> # Amplitude plot for T=0..6,C=4..10
> # Plot 4 zoom-in graphs
> # From the graphics, T=??..??, ??..??, ??..??, ??..??
  # give amplitude spikes above 4 feet. These are
  # determined by left mouse-clicks on the graph, so they
  # are approximate values only.
```