Math 2250-3
Friday, Oct 12, 2001
bases and dimension

```
> with(linalg):
Warning, the name adjoint has been redefined
```

The first problem we will consider here is to find a good basis for the span of 5 vectors in R^4. They are v1:=(1,2,-1,-2), v2:=(2,4,-2,-4), v3:=(0,1,1,0), v4:=(1,4,1,-2), v5:=(1,1, -2,-2). Perhaps they span all of R^5?

Easier in Maple to enter the vectors as rows, and then take transpose:

```
> A:=transpose(matrix(5,4,[1,2,-1,-2,2,4,-2,-4,
  0,1,1,0,1,4,1,-2,1,1,-2,-2]));
```

$$A := \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 4 & 1 & 4 & 1 \\ -1 & -2 & 1 & 1 & -2 \\ -2 & -4 & 0 & -2 & -2 \end{bmatrix}$$

```
> rref(A);
```

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We want to study dependencies of our five vectors. We want to construct a basis for their span. There is some "magic" here:

(1) We know these five vectors MUST be dependent at least to some degreee, even before we do any algebra. Why?

(2) What are the dependencies? What is the actual dimension of the span of the 4 columns? Find a basis.

(3) Here is a related question, more like some of your hw problems from section 4.5: Find a basis for the solution (sub)space of the homogeneous equation Ax=0, for the matrix A above. Explain why the vectors you get which span this subspace, are linearly independent as well.