

Calculations for Ziggy's pendulum and spring

Math 2250-4,

Nov 19, 2001

(1) The pendulum mass is 226 grams, the length (to the ball's center) is 87.5 cm, and the acceleration of gravity is 9.81 m/sec^2 . Use two of these three information pieces to predict the natural period of our pendulum, using our linear pendulum model. Then compare to class experiment. (Ans: about 1.88 seconds per cycle?)

(2) The two mass, two spring system.

Data: Each ball mass is 23 grams. Each spring mass is 9 grams. (Remember, our model assumes massless springs.) The springs are identical, and a mass of 50 grams stretches the spring 15.0 centimeters. (We may check this estimate in class.)

Estimate k (from the data above):

```
[ > k:=solve(k*.15=.05*9.81,k);  
                                     k:=3.2700
```

Time the two natural periods (which we discuss below):

(For the fast one, in my office, I got 100 cycles in 32.7 sseconds. For the slow one I got 100 cycles in 61.3 seconds. What do we get in class?)

Here's the model:

```
[ > with(linalg):  
Warning, the protected names norm and trace have been redefined and unprotected  
[ > A:=matrix(2,2,[-2*k/m, k/m,k/m,-2*k/m]);  
  #this should be the "A" matrix you get for  
  #our two-mass, three-spring system.  
                                     A :=  $\begin{bmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{bmatrix}$   
[ > eigenvects(A);  
                                      $\left[-\frac{k}{m}, 1, \{[1, 1]\}\right], \left[-3\frac{k}{m}, 1, \{[-1, 1]\}\right]$   
[ >
```

Predict the two natural periods from the model:

ANSWER: If you do the model correctly you will come up with natural periods of .304 and .527

seconds. I predict that these periods are too small compared to what actually happened in our experiment, by about 9% and 15%, respectively. What happened?

EXPLANATION: The springs actually have mass, equal to 9 grams each. This is on the same order of magnitude as the ball masses, and causes the actual experiment to run more slowly than our model predicts. In order to be more accurate the total energy of our model must account for the kinetic energy of the springs. You actually have the tools to model this more-complicated situation, using the ideas of total energy discussed in section 5.6, and a "little" Calculus. You can carry out this analysis, assuming that the spring velocity at a point on the spring linearly interpolates the velocity of the wall and mass (or mass and mass) which bounds it. It turns out that this gives the same eigenvectors, but different eigenvalues, namely

$$\lambda_1 = -\frac{k}{m + \frac{5}{6}m_s}$$
$$\lambda_2 = -3\frac{k}{m + \frac{1}{2}m_s}$$

If you use these values, then you get period predictions of .61 and .33 seconds per cycle. How's them apples?

I offer a prestigious Math Department T-shirt to the first student or group of students to submit a correct derivation of this "corrected" model, using total energy ideas. (One T-shirt total). I also offer a 10 midterm points (total) to the successful student or group. I am happy to have further discussion with interested individuals.