HW for Wednesday August 30:
   %1.1:   3, 4, 7, 15, 16, 19, 20, 27, 28, 33, 35, 36
   %1.2:

Remember what a differential equation is?

What is the order of a differential equation?

Don't we get to see some REAL differential equations?

When did the body die?

A **First order differential equation** is any equation which can be written as

$$F\left(x, y, \frac{dy}{dx}\right) = 0$$

For example,

$$y + \left[\frac{dy}{dx}\right]^2 = 1$$

is a differential equation. Often we can rewrite the general D.E. in the more convenient form

$$\frac{dy}{dx} = f(x, y)$$

Can you do this for the example above? No matter how we write the differential equation, the goal is to understand the functions y(x) which make it true. Sometimes we want the particular solution y(x) which satisfies an **initial condition**

$$y(x_0) = y_0$$

Then the corresponding problem is called an **initial value problem**. (The reason for the name "initial condition" is that our independent variable will often be time "t" rather than x, and the initial condition will be the value of the solution function at time t=0, for example an initial population in a population growth problem.)

In section 1.2 the book focuses on the easiest kind first order differential equations to solve, namely ones of the form

$$\frac{dy}{dx} = f(x)$$

Notice the function y(x) does not appear on the right hand side of this D.E so you are looking for antiderivatives of f(x) with respect to x:

$$y = \int f(x)\,dx + C$$

   You got good at antidifferentiation in Calculus!!!! Graviational attraction problems in physics are often of this form, when "t" is the variable. Sometimes more complicated problems also reduce to antidifferentiation.

Example 1. Solve this initial value problem, and then graph the solution function:

$$\frac{dy}{dx} = x - 3$$

$$y(1) = 2$$

Example 2. Solve the initial value problem:

$$\frac{dy}{dx} = y - x$$

$$y(0) = 0$$

Hint: You CANNOT solve this problem by pure antidifferentiation since you can't antidifferentiate the y(x) on the right side if you don't know it yet. We WILL learn an algorithm for solving this sort of DE in section 1.4. For today you may use the fact that the general solution to this differential equation is given by

$$y(x) = x + 1 + C\, e^x$$

Step 1: verify that y(x) does solve the DE.
Step 2: find a value of C in the general solution to solve the IVP.
Step3: graph the solution function.

Slope Fields
And now for something which seems completely different (but isn't). The geometric interpretation of first order differential equations is connected with slope fields. (The use of the word "field" is analogous to how it's used with the prefixes "force", "electric", "magnetic", "wheat".) Slope fields will be discussed in more detail on Monday, and we introduce them now. The differential equation

$$\frac{dy}{dx} = f(x, y)$$

is telling us that the slope of the solution graph at a point (x,y(x)), is given by the function f(x,y). We can use this fact to approximate solutions, especially when exact formulas cannot be found: By hand or computer we can draw a picture of the **slope field** determined by f, i.e. at each point (x,y) assign a slope using the formula f(x,y). The natural way to indicate these slopes is with small line segments, see below. Solutions graphs to the differential equation will be tangent to these slope fields. If there is an initial condition

$$y(x_0) = y_0$$

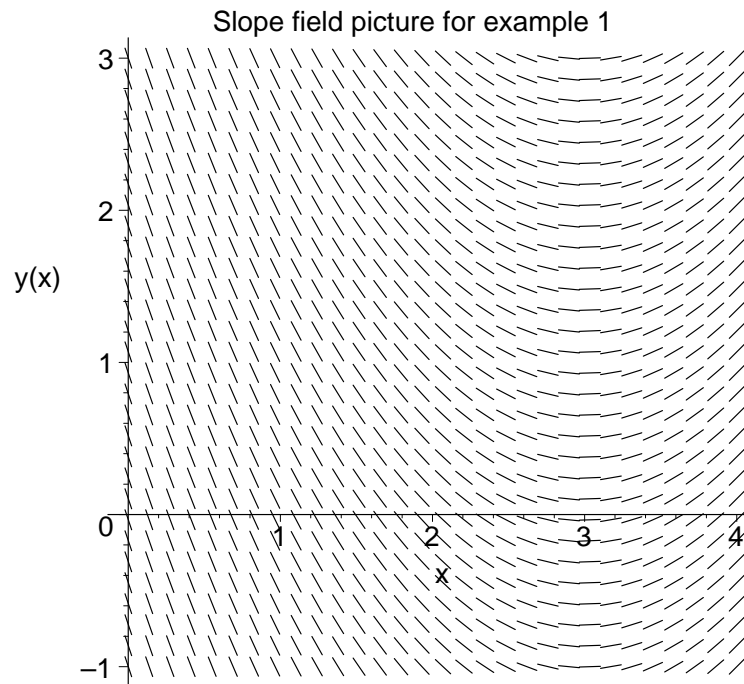then the solution graph will pass through the point

$$[x_0, y_0]$$

Example 1. The initial value problem:

$$\frac{dy}{dx} = x - 3$$

$$y(1) = 2$$

Picture of slope field:

Slope field picture for example 1

Example 2. The initial value problem:
```
> dy/dx = y-x;
  y(0)=0;
```

$$\frac{dy}{dx} = y - x$$

$$y(0) = 0$$

Geometric picture: (Maple commands left in this time.)
```
> with(DEtools):  #differential eqtns command library
>
> deqtn:=diff(y(x),x)=y(x)-x;  #this is example 2
  dsolve({deqtn,y(0)=0},y(x));  #Maple solution
  DEplot(deqtn,y(x),x=-2..2,{[y(0)=0]},y=-3..1,
    arrows=line, color=black,linecolor=black,
    dirgrid=[30,30],
   title='Slope field and solution for example 2');
     #make a picture!
```

$$deqtn := \frac{\partial}{\partial x} y(x) = y(x) - x$$

$$y(x) = x + 1 - \mathbf{e}^x$$

Slope field and solution for example 2
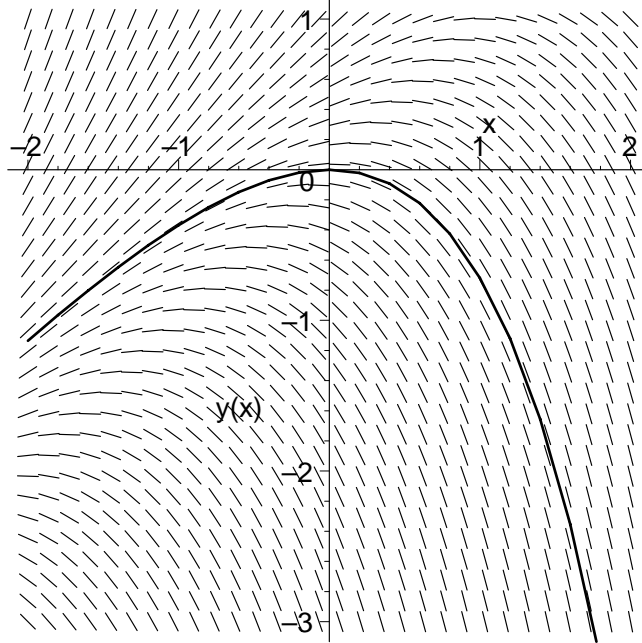
Some real examples!
Example 2 page 13:

Example 4, page 16-17: