MATHEMATICS 2250-3
Ordinary Differential Equations and Linear Algebra
Fall semester 2000

texts: *Differential Equations & Linear Algebra,*
and *Computer Projects* supplement
by C.H. Edwards Jr. and David E. Penney
when: MWF 11:50-12:40
where: JFB 102

instructor: Prof. Nick Korevaar
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office hours: M 2-2:50 p.m., T 12:30-2:30 p.m., W 2-2:50 p.m., F 8:20-9:00 a.m.

course home page: www.math.utah.edu/ ~korevaar/2250fall00.html

prerequisites: Math 1210 and 1220 or the equivalent (first-year Calculus, with a very brief introduction to linear differential equations). In addition you are expected to be familiar with vectors, curves, veclocity (tangent), and acceleration vectors, either from Physics 2210 or Math 2210, or their equivalents.

course outline: This course is an introduction to differential equations, and how they are used to model problems arising in engineering and science. Linear algebra is introduced as a tool for analyzing systems of differential equations, as well as standard linear equations. Computer projects will be assigned to enhance the material.

We will cover most of chapters 1-7 and 10 in the Edwards-Penney text. The course begins with first order differential equations, a subject which you touched on in Calculus. Recall that a differential equation is an equation involving an unknown function and its derivatives, that such equations often arise in science, and that the order of a differential equation is defined to be the highest order derivative occuring in the equation. The goal is to understand the solution functions to differential equations since these functions will be describing the scientific phenomena which led to the differential equation in the first place. In chapters one and two of Edwards-Penney we learn analytic techniques for solving certain first order DE's, the geometric meaning of graphs of solutions, and the numerical techniques for approximating solutions which are motivated by this geometric interpretation. We will carefully study the logistic population growth model from mathematical biology and various velocity-acceleration models from physics.

At this point in the course we will take a four week digression from differential equations, to learn the fundamentals of linear algebra in chapters 3-4. You need a basic understanding of this field of mathematics in order to talk meaningfully about the theory of higher order linear DE's and of systems of linear DE's. Chapter 3 starts out with matrix equations and the Gauss-Jordan method of solution. When you see such equations in high-school algebra you might be thinking of intersecting lines in the plane, or intersecting planes in space, or ways to balance chemical reactions, but the need to understand generally how to solve such equations is pervasive in science. From this concrete beginning we study abstract vector spaces in chapter 4. This abstract theory is useful not only to the understanding of Euclidean space $\mathbf{R}^n$, but it is also the framework which allows us to understand solution spaces to systems of linear differential equations.

In chapter 5 we will study the theory of second order and higher-order linear DE's, and focus on the ones which describe basic mechanical vibrations. You were introduced to these equations in Calculus, and we will treat them more completely now, including a careful study of forced oscillations and resonance.

To succesfully model more complicated physical systems like shaking in multi-story buildings, one needs the framework of linear systems of differential equations. The linear algebra of eigenvalues and eigenvectors is developed in chapter 6 as a tool for studying systems of DE's in chapter 7, where for example eigenvalues and eigenvectors will be related to fundamental modes for mass-spring systems.

Our final course topic is the Laplace transform and its applications to the study of linear DE's, in chapter 10. This "magic" transform takes differential equations and "transforms" them into algebraic equations. You will have to see it in action to appreciate it. You will see, for example, that this method gives a powerful way to study forced oscillations in the physically important cases that the forcing terms are square waves or other discontinuous functions. Electrical engineers often use Laplace transform techniques to understand circuits.

computer projects: There will be three computer projects assigned during the semester, related to the classroom material. They will be written in the software package MAPLE. In addition, you will be asked to use this computer software to check various homework calculations from the linear algebra and Laplace transform sections of the course. There is a Math Deparment Computer Lab at which you all automatically have accounts, and there are other labs around campus where Maple is also available, for example at the College of Engineering. There will tutoring center support for these projects (and for your other homework) as well. There will be introductory sessions to the lab and to Maple, and these will be announced in class. The Math Department Lab is currently located in South Physics 205. More information about the lab can be found at the Math Department web page for undergraduates, www.math.utah.edu/ugrad .

tutoring/help sessions: There will be special help sessions for Math 2250 students, and these will be held on Tuesdays. (Homework will be due on Wednesdays.) Currently the sessions are scheduled as follows, although this may be changed: In JTB 310 from 7:30-9:05 a.m. and from 2:00-4:00 p.m.; in LS 107 from 10:45-11:30 a.m.; in JFB 102 from 11:50-12:40. Students may attend as many or as few of these sessions as they wish.

In addition to the special 2250 sessions, the Math Department Tutoring Center is located in Mine 210, and is open for free tutoring from 8 a.m. to 8 p.m. on M-Th, and from 8 a.m. to 2 p.m. on Friday. Some, but not all of the math tutors welcome questions from Math 2250 students. To see the times and specialities of various tutors, consult the web address www.math.utah.edu/ugrad/tutoring.html .

grading: There will be two midterms, a comprehensive final examination, and homework. Each midterm will count for 20% of your grade, homework (from the book and on the computer) will count for 30%, and the final exam will make up the remaining 30%. The book homework will be assigned daily and collected weekly, on Wednesdays. The three Maple project assignment and collection dates are shown on the syllabus. A homework grader will partially grade your book homework and Maple assignments. The value of carefully working homework problems is that mathematics (like anything) must be practiced and experienced to be learned.

It is the Math Department policy, and mine as well, to grant any withdrawl request until the University deadline of October 20.

ADA statement: The American with Disabilities Act requires that reasonable accomodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.

Tentative Daily Schedule

exam dates fixed, Maple start dates fixed
daily subject matter approximated

W	23 Aug	1.1	modeling and differential equations
F	25 Aug	1.2	integral solutions
M	28 Aug	1.3	direction fields and solution curves
W	30 Aug	1.4	separable equations
		Maple I	*begin* project on chapter 1-2 material
F	1 Sept	1.5	first order linear DE's
M	4 Sept	none	Labor Day
W	6 Sept	2.1	population models
F	8 Sept	2.2	equilibrium solutions and stability
M	11 Sept	2.3	acceleration-velocity models
		Maple I due	*hand in* first Maple project
W	13 Sept	2.4 & 2.6	numerical techniques: Euler and Runge Kutta
F	15 Sept	3.1	introduction to linear systems
M	18 Sept	3.2	matrices and Gaussian elimination
W	20 Sept	3.3	reduced row-echelon form
F	22 Sept	3.4	matrix operations
M	25 Sept	3.5	inverses of matrices
W	27 Sept	3.6	determinants
F	29 Sept	3.6	determinants
M	2 Oct	exam 1	1-3
W	4 Oct	4.1	the vector space $\mathbf{R}^3$
F	6 Oct	none	fall break day
M	9 Oct	4.2	the vector space $\mathbf{R}^n$ and subspaces
W	11 Oct	4.3	linear combinations and linear independence
F	13 Oct	4.4	bases and dimension of vector spaces
M	16 Oct	4.5	general vector spaces
W	18 Oct	5.1	second-order linear differential equations
F	20 Oct	5.2	general solutions of linear equations
M	23 Oct	5.3	homogeneous equations with constant coefficients
W	25 Oct	5.4	mechanical vibrations
F	27 Oct	5.5	undetermined coefficients and variation of parameters
		Maple II	*begin* Tacoma narrows

M	30 Oct	5.5-5.6	and forced oscillations and resonance
W	1 Nov	5.6	continued
F	3 Nov	6.1	introduction to eigenvalues
M	6 Nov	6.2	diagonalization of matrices
		Maple II due	*hand in* second Maple project
W	8 Nov	exam 2	4-6
F	10 Nov	7.1	first order systems and applications
M	13 Nov	7.2	matrices and linear systems
W	15 Nov	7.3	eigenvalue method for linear systems
F	17 Nov	7.3-7.4	& second order systems and mechanical applications
		Maple III	*begin* shaking building
M	20 Nov	7.4	continued
W	22 Nov	none	Semester break day
F	24 Nov	none	Thanksgiving recess
M	27 Nov	10.1	Laplace transforms and inverse transforms
W	29 Nov	10.2	transformation of initial value problems
		Maple III due	*hand in* third Maple project
F	1 Dec	10.3	translation and partial fractions
M	4 Dec	10.4	derivatives, integrals, products of transforms
W	6 Dec	10.4-10.5	& periodic and piecewise continuous forcing functions
F	8 Dec	10.5	continued
Wed	13 Dec	FINAL EXAM	entire course 11:30-1:30