Name..
I.D. number...

Math 2210−4
Exam 2
April 4, 2005

Note to 2210−3 spring 2010: I've added a triple integral at the end of this practice exam.

This exam is closed−book and closed−note. You may use a scientific calculator, but not one which is capable of graphing, integrating or differentiating. **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** There are 100 points possible. The point values for each problem are indicated in the right−hand margin. **Good Luck!**

1a) Compute

$$\int_0^2 \int_0^{x^3} 7\, y \, dy \, dx$$

(10 points)

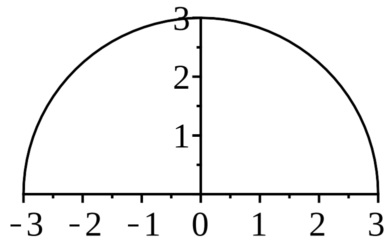
1b) Sketch the region of integration from part (1a), including equations for the boundary curves.

(5 points)

1c) Compute the integral of the same function $f(x,y)=7y$, over the same region, but with the order of integration reversed. (You should get the same answer as in (1a)!)

(10 points)

2) Consider a half−disk laminate, of radius 3 cm, with center at the origin as indicated below. Suppose this laminate has constant density $\frac{1}{6} \frac{gm}{cm^2}$.



2a) Find the mass of the laminate, including units. You may use geometry to avoid computing an integral.

(5)

2b) Because the half−disk laminate and its density are symmetric with respect to the y−axis, we know that the x−coordinate of the center of mass is zero. Find the y−coordinate of the center of mass, including units. You may do this computation using either polar or rectangular coordinates; it should work out nicely either way.

(15 points)

3) Consider the following constrained optimization problem: We wish to manufacture rectangular, open−topped boxes with volume 800 cubic inches. Material for the sides costs 5 cents per square inch, whereas the material for the bottom costs 8 cents per square inch.

3a) Derive a formula for the cost C of producing one box, depending on the dimensions x,y,z.

(5 points)

3b) Given that volume V=xyz is constrained to be 800 cubic inches, what dimensions minimize the cost C of each box? I would recommend that you solve this problem by first using the volume constraint to eliminate one of the variables from the cost function. Alternately, you may use Lagrange multipliers. No second derivative test is necessary.

(15 points)

4) In this problem we will consider the function $f(x, y) = e^{2x}(2 + \sin(y))$.

4a) Compute the gradient of f(x,y). Then verify that its value at (0,0) is the vector $< 4, 1 >$.

(5 points)

4b) In what unit direction is f(x,y) increasing most rapidly at the origin? What is the rate of change of f in this direction?

(5 points)

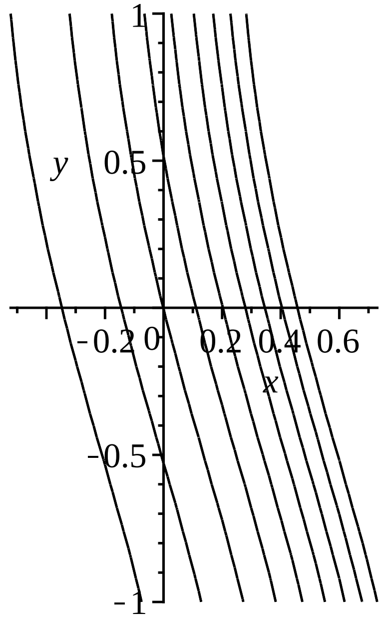
4c) Use the differential (or tangent) approximation to approximate f(0.1, −0.2), using f(0,0)=2.

(5 points)

4d) Compute the directional derivative $D_u f(0, 0)$, where **u** is the unit vector in the same direction as the vector $<-1, 1>$.

(5 points)

Here is a picture of some level curves of f(x,y). Each successive level curve increments f(x,y) by 0.5, moving left to right.



4e) Use the level curve picture and an index card ruler (or a piece of the last page of the test) to estimate the same directional derivative you just computed exactly in (4d). Make sure to show/explain your work! (Your estimate may only be within 10 or 20% of the exact value, because you're not working on a very magnified scale.)

(5 points)

4f) Let **r**(t) be the vector−valued function **r**(t) = < $-e^{2t}$, $-\sin(2\,t)$ >. Use the multivariable chain rule to compute the t−derivative of the composition f(**r**(t)), at t = 0.

(10 points)

5) Find the volume of the region inside the cylinder $x^2 + y^2 = 9$, above the x − y plane, and inside the sphere $x^2 + y^2 + z^2 = 25$. Set up you integral as a double integral using rectangular coordinates and polar coordinates. Also set it up as a triple integral using rectangular coordinates and cylindrical coordinates. Evaluate the integral using whichever of these seems easiest to evaluate.