Math 1210-3

Monday March 8

- quickly go over the differential approximation example at the end of Friday's notes ~ these concepts relate to the vector calculus integration in Chapter 14, just as differentials are used in 1-d integrals.

so we have time for

§12.8 Max-min problems
for fcns of several variables

## Notation

Let $f: D \to \mathbb{R}$

$f(x,y)$

$f(x,y,z)$ etc.

$(\vec{p}_0)$ is global max. value
if $f(p_0) \geq f(p)$ $\forall p \in D$

global min. value
if $f(p_0) \leq f(p)$ $\forall p \in D$

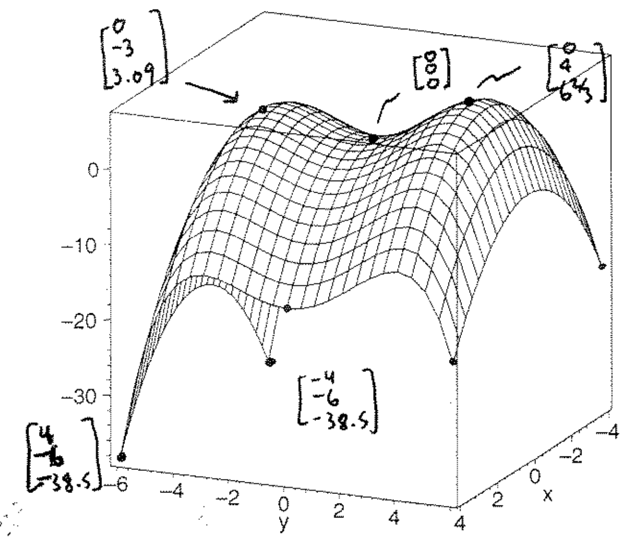
$f(\vec{p}_0)$ is a local max or min value

if there is a neighborhood $N$ of $\vec{p}_0$ so that

$f(\vec{p}_0)$ is the max or min value of $f$ is $D \cap N$

$z = f(x,y)$

$z = \frac{3}{4}y^2 + \frac{1}{24}y^3 - \frac{1}{32}y^4 - x^2$




Domain $-4 \leq x \leq 4$
$-6 \leq y \leq 4$

i.e. a closed rectangular domain
contains its boundary

**Theorem**: If $D$ is a closed and bounded domain

closed: includes its boundary

bounded: all points are within some fixed finite distance of origin

And if $f$ is continuous at all points of $D$

Then $f$ attains its global maximum and minimum values. This either happens at boundary points or interior points. If the extreme value is $f$ of an interior point $p_0$ then either

(a) $\nabla f(p_0) = 0$

or (b) $f$ is not diff'ble at $p_0$

Example 1) By inspection above

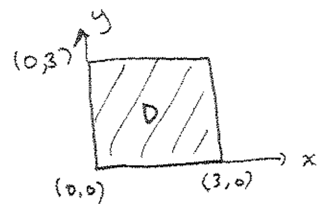
(1) Find all global extrema

(2) Find some local max & min values which are not global extrema

(3) What's interesting about $(0,0,0)$?

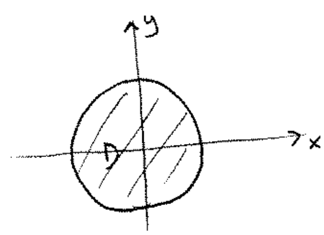
Example 2

$$D = \left\{ (x,y) \;\middle|\; \begin{matrix} 0 \le x \le 3 \\ 0 \le y \le 3 \end{matrix} \right\}$$




Find the global max and min values of $f(x,y) = 3x^2 - 6x + 2y^2 - 8y + 13$

Example 3 Let $D = \{(x,y) \mid x^2+y^2 \le 1\}$




Find the global max and min values of $f(x,y) = x^2 + y^2 + 3xy$

**second derivative test, for $f(x,y)$:**

If $\nabla f(p) = 0$

Compute the Hessian matrix $\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$

and let $D = f_{xx} f_{yy} - f_{xy}^2$ be its determinant at $p$

If

(1) $f_{xx} > 0$ AND $D > 0$ $\quad$ $f$ is concave up $\rightarrow$ $f(p)$ local min

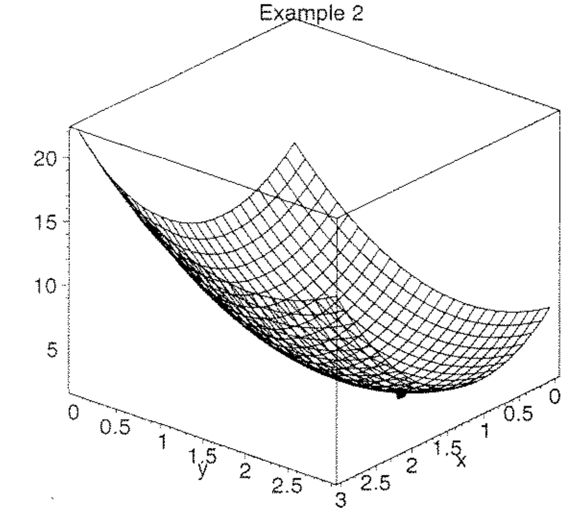
(2) $f_{xx} < 0$ AND $D > 0$ $\quad$ $f$ is CD $\rightarrow$ $f(p)$ local max

(3) $D < 0$ $\quad$ $f(p)$ is a saddle point $\quad$ neither local max or min

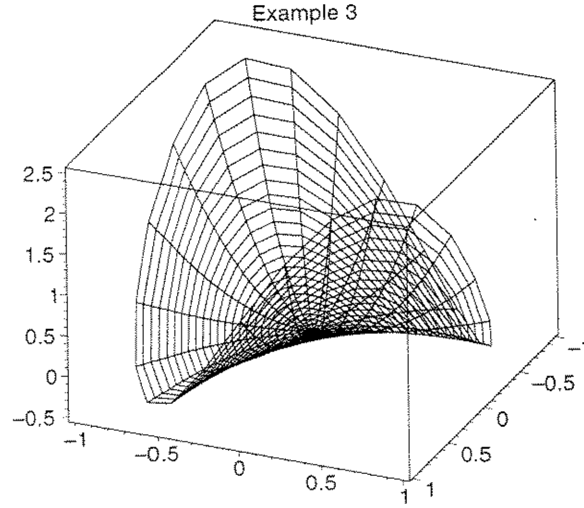
(4) $D = 0$ $\quad$ can't say anything w/o more work

Check Examples 2 & 3 critical pts
with $2^{nd}$ deriv test:

Example 2

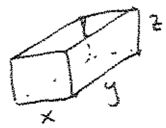



Example 3



## Applications (more in HW!)

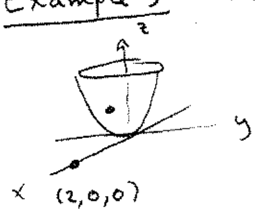
### Example 4




open-topped box made of cardboard
Volume to be $4\ m^3$
What choices of $x, y, z$ minimize required surface area of cardboard?

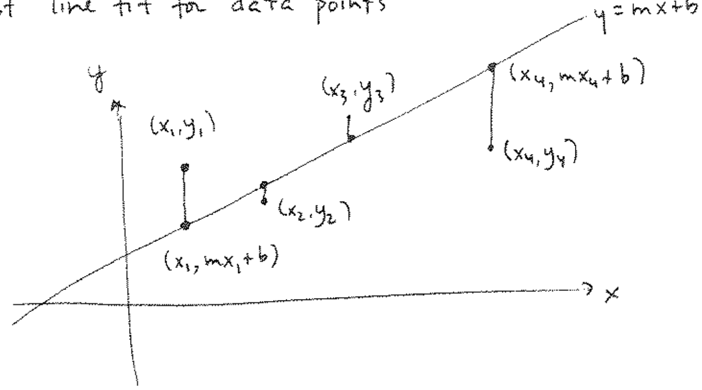
### Example 5

Find the nearest point on the paraboloid $z = x^2 + y^2$, to $P = (2,0,0)$



Example 6 : "Linear regression" is an extremely important statistics ~~proble~~ tool, which is really a solution to a max-min problem:

"best" line fit for n data points




$f(m,b)$ = sum of squared vertical deviations

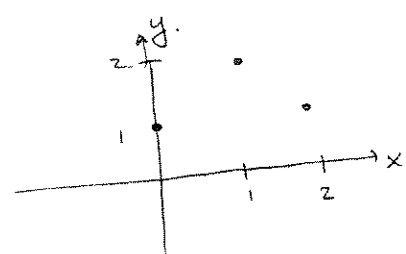
$$= \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

Find $m, b$ to minimize $f(m,b)$. Then $y = mx+b$ is called the linear regression line.

In 12.8.30 you show the critical pt $(m_0, b_0)$ satisfies

$$\boxed{\begin{aligned} m\left(\sum x_i^2\right) + b\left(\sum x_i\right) &= \sum x_i y_i \\ m \sum x_i + nb &= \sum y_i \end{aligned}}$$

Example : Find best line fit for $\{(0,1), (1,2), (2,1)\}$




$n = 3$

$\sum x_i =$

$\sum y_i =$

$\sum x_i y_i =$

$\sum x_i^2 =$