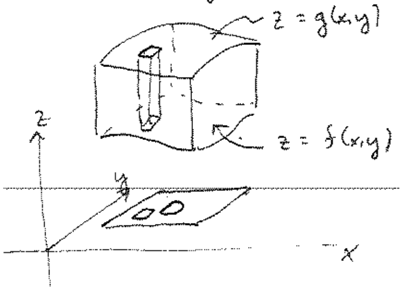
Math 2210-3
Friday March 19

§13.5 applications of double integrals

① Volume of a region between 2 graphs



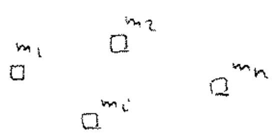
$dV = (g(x,y) - f(x,y))\, dA$

$$V = \iint_D g(x,y) - f(x,y)\, dA$$

Example: you've done lots of examples where the lower graph was just $z=0$.
even one or two (e.g. ball volume) between two graphs.

② Mass of lamina

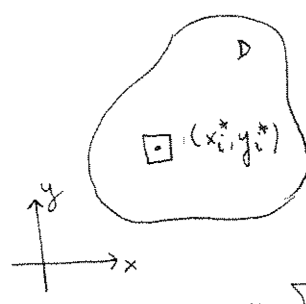
Discrete.



total mass

$$m = \sum_{i=1}^{n} m_i$$

lamina



$\delta(x,y)$ = density fcn
units mass/area

$$m \approx \sum_i \underbrace{\delta(x_i^*, y_i^*)}_{(mass/area)} \underbrace{\Delta A_i}_{(area)}$$ Riemann sum

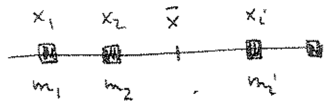
$$\boxed{m = \iint_D \delta(x,y)\, dA}$$

Discrete

) centers of mass

on a line (teeter totter)

$\bar{x}$ satisfies the no net torque condition



et
rque
at $\bar{x}$:
oportional
to

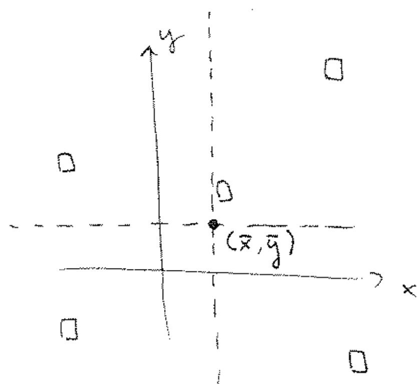
$$\sum_i m_i(x_i - \bar{x}) = 0$$

i.e.

$$\sum_i m_i x_i - \bar{x}\sum m_i = 0$$

$$\boxed{\bar{x} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_i x_i}{m}}$$

masses in a plane have center of mass $(\bar{x}, \bar{y})$ (balance point)



$$\sum m_i(x_i - \bar{x}) = 0$$

$$\& \sum_i m_i(y_i - \bar{y}) = 0$$

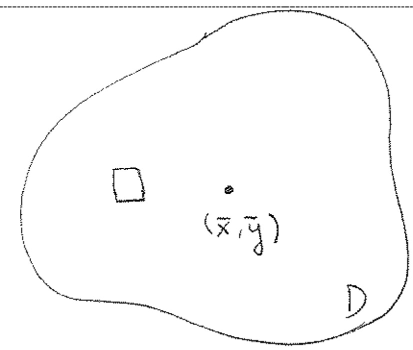
moment wrt y-axis!

So $\boxed{\bar{x} = \frac{\sum m_i x_i}{m} = \frac{M_y}{m}}$

moment wrt x-axis

$\boxed{\bar{y} = \frac{\sum m_i y_i}{m} = \frac{M_x}{m}}$

Lamina



$(\bar{x}, \bar{y})$ defined by

$$\iint_D (x - \bar{x})\,\delta(x,y)\,dA = 0$$

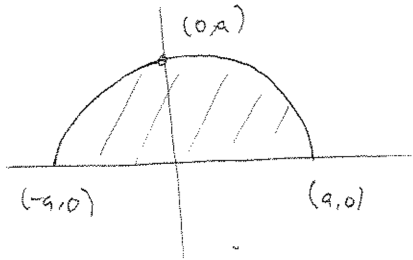
$$\iint_D (y - \bar{y})\,\delta(x,y)\,dA = 0$$

yields

$$\boxed{\bar{x} = \frac{\iint_D x\,\delta(x,y)\,dA}{m}} \quad \left(= \frac{M_y}{m}\right)$$

$$\boxed{\bar{y} = \frac{\iint_D y\,\delta(x,y)\,dA}{m}} \quad \left(= \frac{M_x}{m}\right)$$

Example: Consider a half disk with constant density δ : $x^2+y^2 \le a^2$
$y \geq 0$



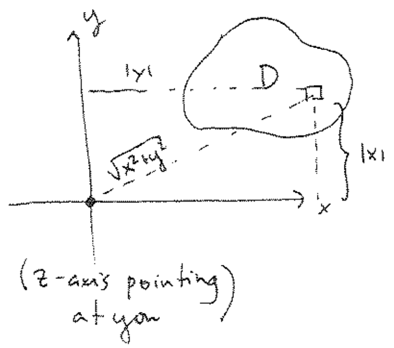
- explain why $\bar{x} = 0$

- find $\bar{y}$

ans $\bar{y} = \frac{4}{3\pi} a \approx .424a$

④ Moments of inertia I_x, I_y, I_z

(Used to calculate kinetic energy when object is rotated about an axis; $KE = \frac{1}{2} I \omega^2$



(z-axis pointing at you)

$$I_x = \iint_D y^2 \, \delta(x,y) \, dA \qquad \left(\text{Note } KE = \iint_D \tfrac{1}{2}(y\omega)^2 \delta(x,y)\, dA = \tfrac{1}{2} I_x \omega^2 \right)$$

$$I_y = \iint_D x^2 \, \delta(x,y) \, dA$$

$$I_z = \iint_D (x^2+y^2) \, \delta(x,y) \, dA \qquad (= I_x + I_y \, !)$$

<u>example</u> : Find I_x for the half disk in page 3 example

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> assume(a,positive):
  simplify(int(int(y^2,y=0..sqrt(a^2-x^2)),x=-a..a),radical);
```

Int table #62

$$\frac{a\sim^4 \pi}{8}$$

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> int(int(r^3*sin(theta)^2,theta=0..Pi),r=0..a);
```

Int table #20

$$\frac{a\sim^4 \pi}{8}$$