Math 2210-3
Wednesday March 10

HW for March 17

| | |
|---|---|
| 12.8 | 19, 20, 22, 24, 30, 31 |
| 12.9 | 1, 3, 5, 9, 11, 14, 16 |
| 13.1 | 1, 5, 8, 9, 10, 16 |
| 13.2 | 6, 7, 13, 14, 25, 26 |

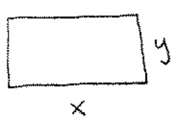
- Finish Monday notes, which show how to modify Calc 1 methods for minimizing functions with constraints, to the case of more variables.

- The "method of least squares", from Monday.

Then to 12.9: <u>Lagrange's method for constrained max-min problems</u>

Remember our (my) favorite Calculus problem:

Find the rectangle of area one which has minimum perimeter




minimize $P(x,y) = 2x + 2y$

subject to $xy = 1 = A(x,y)$

old way: $y = \frac{1}{x}$

minimize $f(x) = 2x + \frac{2}{x}$

critical pts: $f'(x) = 0 = 2 - \frac{2}{x^2}$

$2 = \frac{2}{x^2}$

$x^2 = 1$

$x = 1 \Rightarrow y = 1$

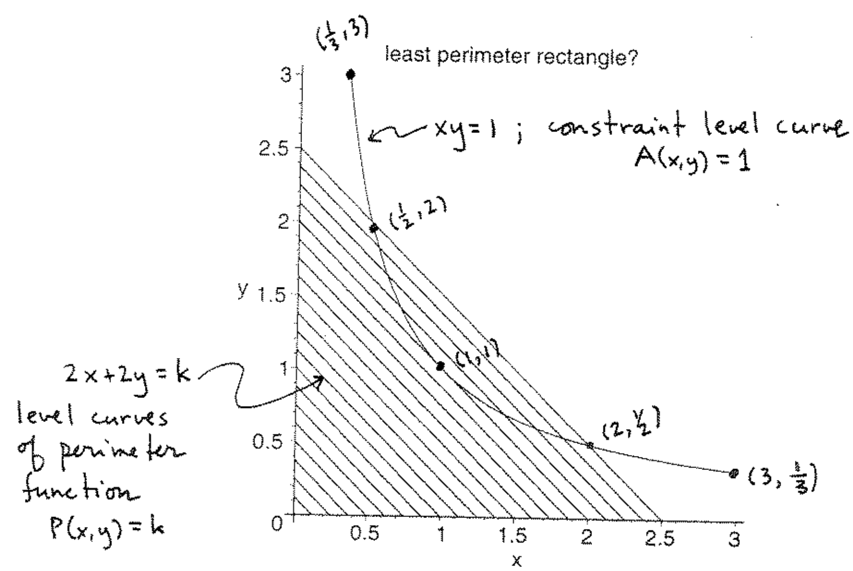
square!

What do you notice about

$\nabla P$ and $\nabla A$

and the point $(1,1)$?

Explain why this must be so!




This idea leads to Lagrange's neat way of solving constrained optimization problems...

## Lagrange multipliers

If $F(p_0)$ is a maximum or minimum value of $F(x,y)$ on the level curve $G(x,y)=k$
$F(x,y,z)$ level surface $G(x,y,z)=k$
etc. etc.

Then $\nabla F(p_0) = \lambda \nabla G(p_0)$

for some scalar $\lambda$, called the Lagrange multiplier.

example 1 Minimize $P(x,y) = 2x + 2y$
subject to $A(x,y) = xy = 1$
using Lagrange multipliers, i.e. redo page 1 problem this new way.

example 2 (from §12.8 # 13)(hw due today)
original problem was to find max & min values
of $f(x,y) = x^2 - y^2 + 1$ in the disk $\{(x,y) \text{ s.t. } x^2+y^2 \le 1\}$.
the only critical point was $(0,0)$, but it's a saddle.
So max and min occur on the circle $g(x,y) = x^2+y^2 = 1$. ← constraint
Now, use Lagrange!

Why Lagrange multipliers works:
Let $F(p_0)$ be a max or min value of $F(p)$ on the level surface $G(p) = \text{const}$.
Let $\vec{r}(t)$ be any curve (with range) on the level surface $G(p) = \text{const}$, i.e. $G(\vec{r}(t)) = \text{const}$

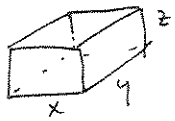
If $\vec{r}(t_0) = \vec{p}_0$ and $F(\vec{p}_0)$ is a max or min value
then $F(\vec{r}(t_0))$ is a local max or min for $F(\vec{r}(t))$

$\Rightarrow (F \circ \vec{r})'(t_0) = 0 = \nabla F(\vec{p}_0) \cdot \vec{r}\,'(t_0)$

Deduce $\nabla F(\vec{p}_0) \perp$ level $\left(\begin{smallmatrix}\text{curve or}\\ \text{surface}\end{smallmatrix}\right)$. But so is $\nabla G(\vec{p}_0)$. $\Rightarrow \nabla F(\vec{p}_0) = \lambda \nabla G(\vec{p}_0)$ ∎

Example 3

Rework Example 4 Monday's notes with Lagrange multipliers:




$V = 4 = xyz$

minimize $A = 2xz + 2yz + xy$

it's going to be easier!