Math 2210-3
Wednesday Jan 20

§11.3 cont'd.

Last week:

$\mathbb{R}^3$

vectors

geometric

algebraic

$+$

scalar mult

dot product

definition : $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ $\quad (\vec{a} = \langle a_1, a_2, a_3\rangle,\ \vec{b} = \langle b_1, b_2, b_3\rangle)$

geometric formula: $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

- continue discussion, page 2 Friday notes. & page 3

Applications of dot product

(A) Use geometric formula to find $\underline{\cos\theta}$ (or θ); did examples Fri. Also, see HW

(B) Planes in $\mathbb{R}^3$:

Why the graph of $ax+by+cz=d$ is a plane in $\mathbb{R}^3$ and the importance of $\langle a,b,c\rangle$:

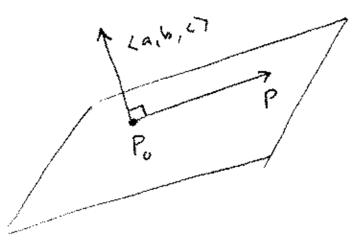
Let $P_0 = (x_0, y_0, z_0)$ be a soltn of our eqtn

Let $P = (x, y, z)$ be any soltn of our eqtn

$$\left.\begin{array}{l} ax_0 + by_0 + cz_0 = d \\ ax + by + cz = d \end{array}\right\} \quad ax+by+cz = ax_0+by_0+cz_0$$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$\langle a, b, c\rangle \cdot \underbrace{\langle (x-x_0), y-y_0, z-z_0\rangle}_{\overrightarrow{P_0P}} = 0$$



geometrically describes a plane through P_0 with normal ($\perp$) vector $\langle a,b,c\rangle$!

i.e. the plane is the set of all $P=(x,y,z)$ so that $\langle a,b,c\rangle \perp \overrightarrow{P_0P}$!

1 Example

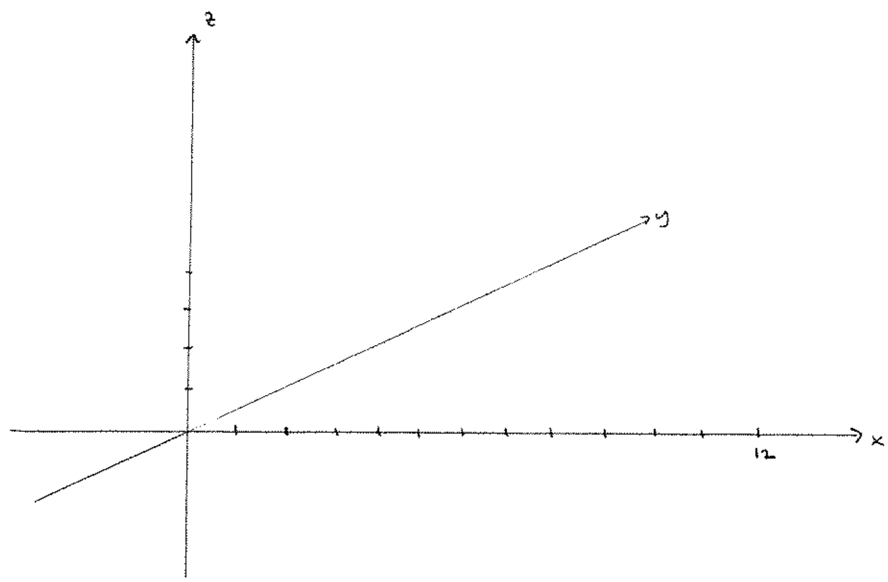
(a) Sketch the plane

$$x + 3z = 12 \quad \text{in } x\text{-}y\text{-}z \text{ space.}$$

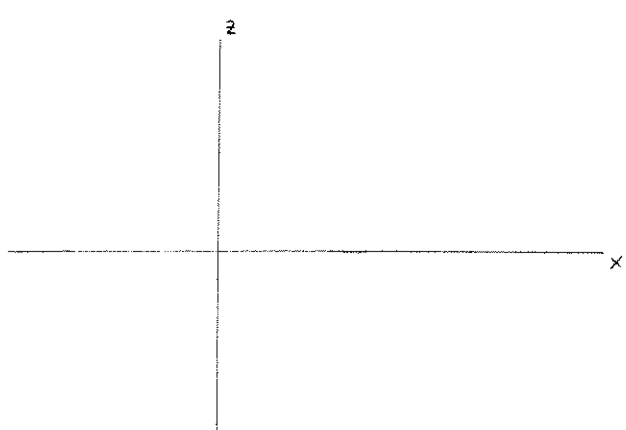
Include a normal vector

(b) Add the plane $x = 8$

(c) What is the (smaller of the two) angle(s) between these two planes?
(notice it's the angle between the two plane normal vectors!)



cross section in a plane ⊥ to both planes:



Remark: you could redo lines in $\mathbb{R}^2$ with the dot product. Also, $(n-1)$-dimensional hyperplanes in $\mathbb{R}^n$

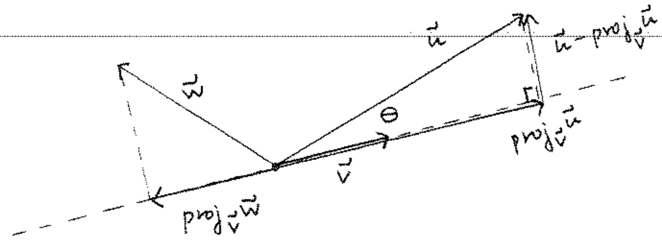
Application

(C) Projections

the (vector) projection of $\vec{u}$ onto $\vec{v}$ $\text{pr}_{\vec{v}}\vec{u}$ (or $\text{proj}_{\vec{v}}\vec{u}$)

is the unique scalar multiple of $\vec{v}$ so that $\vec{u} - \text{proj}_{\vec{v}}\vec{u}$ is $\perp$ to $\vec{v}$:

$$\text{proj}_{\vec{v}}\vec{u} = \underbrace{|\vec{u}|\cos\theta}_{\text{dist}} \underbrace{\frac{\vec{v}}{|\vec{v}|}}_{\text{unit vect in } \vec{v} \text{ dir}}$$



$$= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\,\frac{\vec{v}}{|\vec{v}|}$$

$$\boxed{\begin{aligned} \text{proj}_{\vec{v}}\vec{u} &= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\,\vec{v} \\ \text{comp}_{\vec{v}}\vec{u} &= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} \end{aligned}}$$

the scalar projection of $\vec{u}$ onto $\vec{v}$ (also called the component of $\vec{u}$ in the $\vec{v}$ direction) $\text{comp}_{\vec{v}}\vec{u}$

is $|\vec{u}|\cos\theta$,

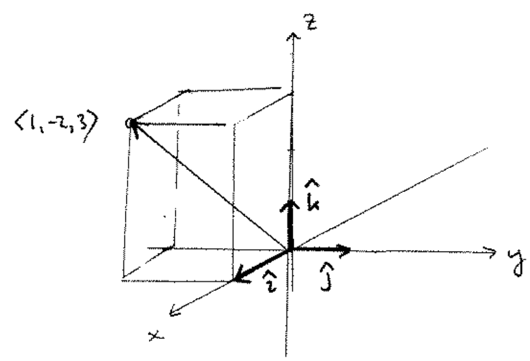
i.e. the signed length of $\text{proj}_{\vec{v}}\vec{u}$

(2) Example

$\vec{u} = \langle 1, -2, 3\rangle$

What are the (vector) projections of $\vec{u}$ in the $\hat{\imath}, \hat{\jmath}, \hat{k}$ directions?

What are the corresponding components?



Example

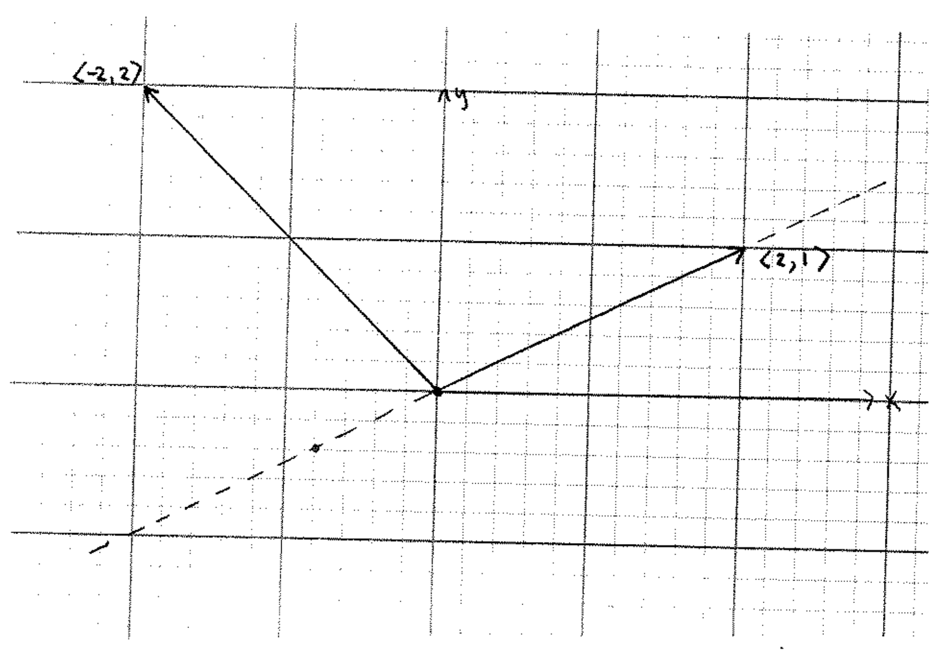
(3) Let $\vec{u} = \langle -2, 2\rangle$

$\vec{v} = \langle 2, 1\rangle$

Compute

$\text{proj}_{\vec{v}}\vec{u}$, $\text{comp}_{\vec{v}}\vec{u}$.

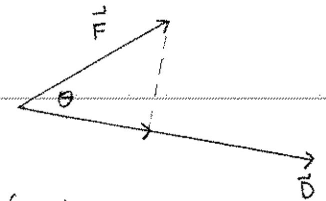
Illustrate.



Application

D Work :

If $\vec{F}$ is a constant force field
and an object is displaced by the vector $\vec{D}$



Then work W is defined
by $W = \underbrace{(\text{comp}_{\vec{D}} \vec{F})}_{\frac{\vec{F} \cdot \vec{D}}{|\vec{D}|}} |\vec{D}|$ (how we did work in 1210-1220, using scalar computations)

so $\boxed{W = \vec{F} \cdot \vec{D}}$ This is the work done by the field,
which is opposite the work done by the object

Example 4

A 10 kg mass is subject to
a force of $10g = 98$ Newtons,
pointing vertically down (from gravity)

i.e. $\vec{F} = \langle 0, 0, -98 \rangle$

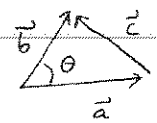
How much work is done (by the field)
in moving ~~the~~ object from $\langle 1, -2, 3 \rangle$ to $\langle 100, 0, 103 \rangle$?
(distance measured in meters)

Homework for Wed. Jan 27

As always, circled problems hand in, others recommended

1. Verify the law of cosines

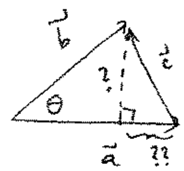
$$|\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$$



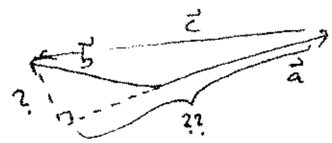
using trig, as indicated.

(a) $\theta = 0, \pi/2, \pi$. (These are special cases; $\theta = \pi/2$ is just Pyth, so you don't need to redo it, after last HW)

(b) $0 < \theta < \pi/2$. Label so that $|\vec{b}| \le |\vec{a}|$. Find lengths ?, ?? from trig. Then apply Pyth. thm.



(c) $\pi/2 < \theta < \pi$. Follow same steps as in (b):



Book problems:

11.3 24, 25, 27, 37, 43, 54, 61, 64, 65, 69, 73, 76

in 25 & 27 draw pictures and label the vectors and their projections

11.4 3, 5, 10, 11, 17, 21, 25, 31, 32

11.5 3, 9, 13a, b, 19, 32, 34