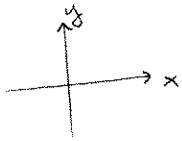
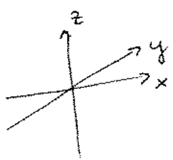
Math 2210-3
Monday 11 January
§ 11.1 3-space, $\mathbb{R}^3$

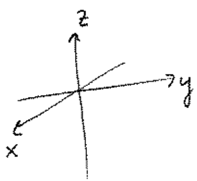
1210-1220: $\mathbb{R}^2$
"x-y plane"



2210: mostly $\mathbb{R}^3$
"x-y-z space"

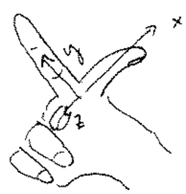


or



all axes are chosen perpendicular to eachother.

we choose right-handed ways of labeling our three directions (as opposed to left-handed)



or

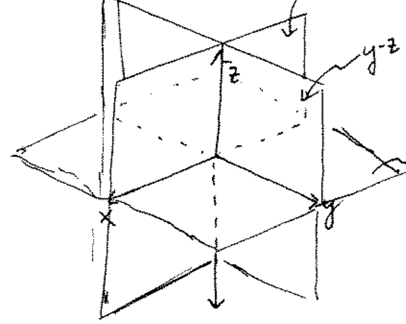
curling from x to y axis.

(We had an analogous convention in $\mathbb{R}^2$)

If $P = (x_1, y_1, z_1)$ in $\mathbb{R}^3$
that means we have displaced from the origin by amts

x_1 in x-dir
y_1 in y-dir
z_1 in z dir.

eight octants in 3-space.
usually only specify 1st octant → where
$x > 0$
$y > 0$
$z > 0$
all hold.



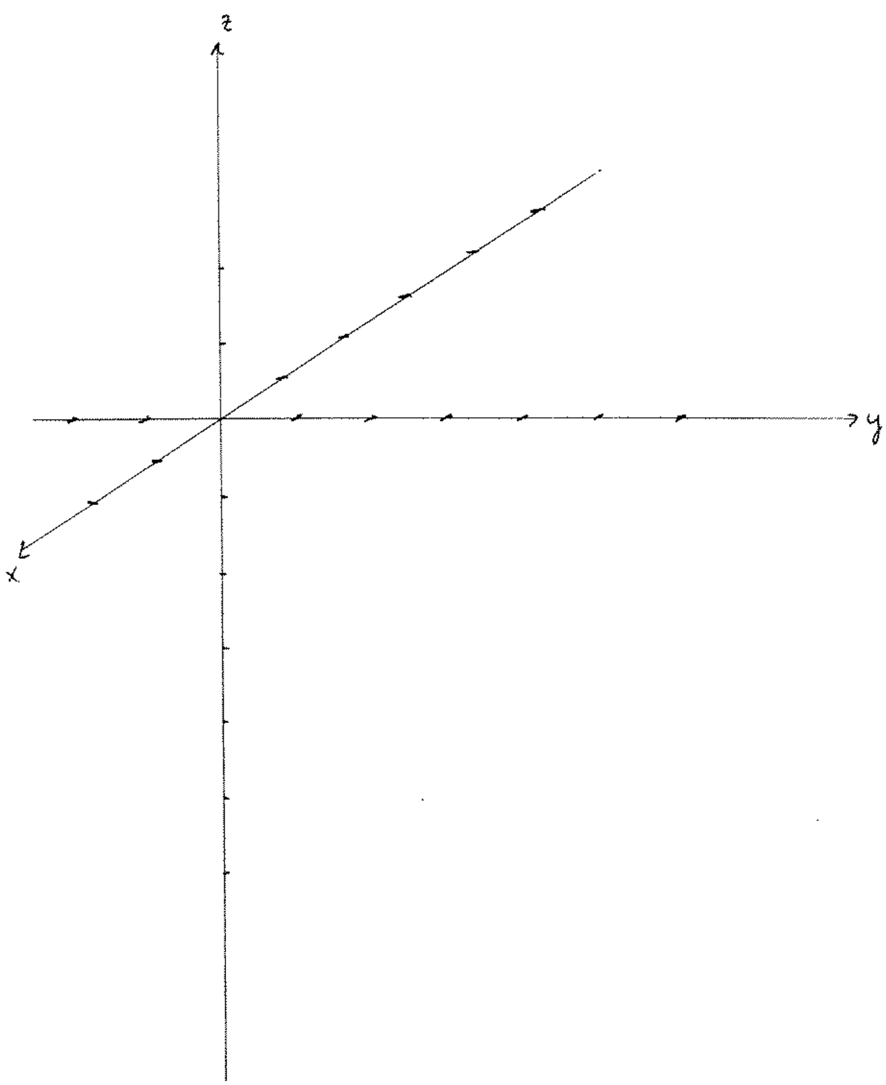
Examples

) plot the point
$P = (-4, 3, -5)$
as well as the rectangular coord box which has P and the origin as opposite corners.

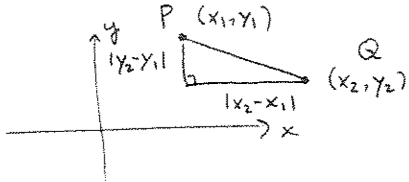
② Use equalities and inequalities to specify

(a) the region inside the box
(b) two different rectangular faces of the box
(c) two different edges of the box.

③ Try to figure out the straight line distance from the origin to P.

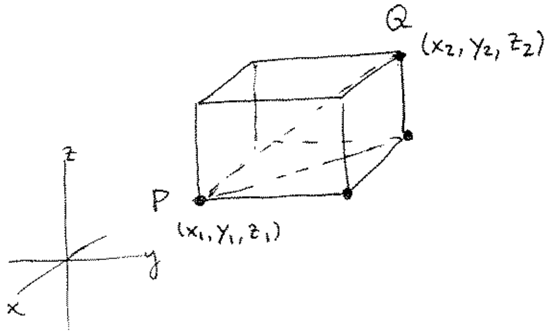


Distance formulas



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Pythagorean Thm!



$d = ?$ Use Pythag twice!

Example (4): Identify and sketch the points which satisfy

$$x^2 + y^2 + z^2 - 10x - 8y - 12z + 68 = 0$$

(We call this "graphing the equation")

hint: complete the square first.

Examples

5. graph the equation $x^2 + y^2 = 1$
6. graph the region $1 \le x^2 + y^2 \le 4$
7. graph the plane $x + 2y = 4$
8. graph (a piece of) the plane
$$x + 2y + 3z = 6$$
9. graph (a piece of) the plane
$$x + 2y + 3z = 0$$
10. graph the equation $z = y^2$

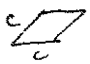
Math 2210-3

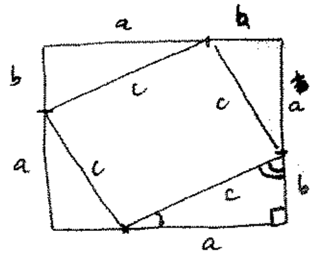
First HW assignment, due Wed Jan 20

Circled problems are to be handed in, others (not circled) are recommended if you want extra practice but won't be graded

(this assignment covers more than 1 week of material, so is somewhat large.)

1. The Pythagorean Theorem says that for a right Δ, with legs a, b & hypotenuse c, $\quad c^2 = a^2+b^2$

   Use the following diagram (by computing the area of the $(a+b)\times(a+b)$ square two ways) to prove the Pythagorean Thm. Hint: first show that the inside  is a square, using the fact that the sum of angles in a triangle is $180°$



2. Consider the point $P=(1,-2,3)$.

   (a) Draw the x-y-z axes as on page 2 of today's (1/10) notes, and then draw the coordinate box for P, as we did on page 2. (So P & the origin are opposite vertices.)

   (b) Use inequalities to specify the region inside the box

   (c) Use inequalities and the equality $x=1$ to specify the "front" face of the box

   (d) Use equalities and inequalities to specify (separately) the three edges which contain the point $(1,-2,3)=P$

   (e) How far is it from P to the origin?

   (f) " " " " to the x-y plane?

   (g) " " " " to the x-axis?

3. Sketch pieces of the following surfaces or regions which satisfy the given eqtns or inequalities

   (a) $x^2+y^2+z^2=9$

   (b) $x^2+y^2+z^2\le 9$

   (c) $x^2+y^2=4$

   (d) $x^2+y^2\le 4$

11.1: 13, 14, 17 22 25, 28 31 ← in 31 also sketch this helix (which lies on the cylinder $x^2+y^2=4$)
38

11.2 2 3 4 7, 15, 17 27

11.3 1 ac e, 2 b df, 3b 4c 8 17 24 25 27 In 25 & 27 draw pictures with the vectors & projections
37 43 54 (this is called the parallelogram identity. why?) 61, 64 65 69 73 76