Math 2210-3
Monday Feb. 8 11.7-11.8

Recall that for a parametric curve with position vector $\vec{r}(t)$, domain $a \le t \le b$
positive speed $\|\vec{r}'(t)\| > 0$
$\quad = v(t)$

- arclength function $s(t) = \int_a^t \|\vec{r}'(\tau)\| d\tau$

$$\frac{ds}{dt} = \|\vec{r}'(t)\| = \text{speed } v(t)$$

$\frac{ds}{dt} > 0$ for $a \le t \le b$ implies inverse function $t(s)$ exists, $0 \le s \le L = \int_a^b v(\tau)d\tau$

- unit tangent vector $\vec{T} := \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{1}{v}\vec{r}'(t)$

$$\vec{T}\cdot\vec{T} \equiv 1 \Rightarrow 2\vec{T}\cdot\vec{T}' \equiv 0 \Rightarrow \vec{T}'(t) \perp \vec{T}.$$

- curvature $\kappa := \|\frac{d\vec{T}}{ds}\| = \|\vec{T}'(t)\frac{dt}{ds}\| = \frac{1}{v}\|\vec{T}'(t)\|$
- unit normal vector $\vec{N} := \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|} = \frac{1}{\kappa v}\vec{T}'(t)$

we checked that curvature of a circle of radius R is $\frac{1}{R}$ & curvature of a straight line is 0.

the circle of curvature at a point $\vec{r}(t_0)$ is the circle which has same tangent & normal vectors as does the curve parameterized by $\vec{r}(t)$, at $\vec{r}(t_0)$. Also, same curvature.

Do Exercise 3 on Friday's notes.

- from $\vec{r}'(t) = v\vec{T}$

deduced roller coaster equation

$$\vec{r}''(t) = v'\vec{T} + v\vec{T}'(t)$$

$$\boxed{\vec{r}''(t) = v'(t)\vec{T} + \kappa v^2\vec{N}}$$

generalizes what you learn in physics – that for circular motion, centripetal acceleration is $\frac{v^2}{R}$.

Do exercise 5 on Friday's notes

Remark: There are magic formulas to compute κ in the text & hw.
They're not magic at all, you get them by crossing $\vec{T}$ with the RCE:

$$\underbrace{\vec{T}}_{\frac{1}{v}\vec{r}'(t)} \times \vec{r}''(t) = \vec{T} \times (v'(t)\vec{T} + \kappa v^2\vec{N})$$
$$= \underbrace{\vec{T}\times(v'(t)\vec{T})}_{0} + \kappa v^2\vec{B}$$

$\vec{B} : \vec{T}\times\vec{N}$ is called the binormal vector
it's a unit vector $\perp$ to $\vec{T}$ & $\vec{N}$.

take $\|\cdot\|$: $\frac{1}{v}\|\vec{r}'\times\vec{r}''\| = \kappa v^2 \longrightarrow \kappa = \frac{1}{v^3}\|\vec{r}'\times\vec{r}''\| = \boxed{\frac{\|\vec{r}'\times\vec{r}''\|}{\|\vec{r}'\|^3} = \kappa}$

if $\vec{r}(t) = \langle x(t), y(t), 0\rangle$ is in the x-y plane this becomes

$$\boxed{\kappa = \frac{|x'y'' - y''x'|}{(x'^2 + y'^2)^{3/2}}}$$

§11.8 Quadric surfaces, cylinders, surfaces of revolution

Quadric surfaces: surfaces satisfying a quadratic equation in x, y, z variables

cylinders: surfaces with implicit equation only depending on two variables.

surfaces of revolution: surface obtained by rotating a curve about a line "axis"

- We can visualize these surfaces by understanding their intersections (called "traces") in various planes; the 3 most important planes to use are the coord planes: x-y plane ($z=0$), x-z plane ($y=0$), y-z plane ($x=0$)

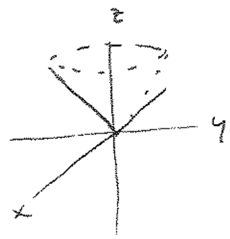
Example 1

sketch $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{1} = 1$

Example 2

sketch the "vertical cylinder" $y = \cos x$

Example 3



take the line $z = x$ in x-z plane $(x > 0)$
& rotate about the z-axis to make a cone.

Find an eqtn for the cone.

Example 4 $z = x^2 + y^2$

Example 5 $z = x^2 - y^2$

Example 6 $x^2 + y^2 - z^2 = 1$

then try
$x^2 + y^2 - z^2 = -1$
if you have time.