Math 2210-3
Wednesday Feb. 24 §12.3

HW for Wed March 9
12.3 1, 2, 3, 7, 9, 10, 21, 35, 36, 37, 38
12.4 2, 9, 11, 12, 15, 17
12.5 1, 9, 14, 15, 17, 19, 20, 24, 25, 31

limits and continuity

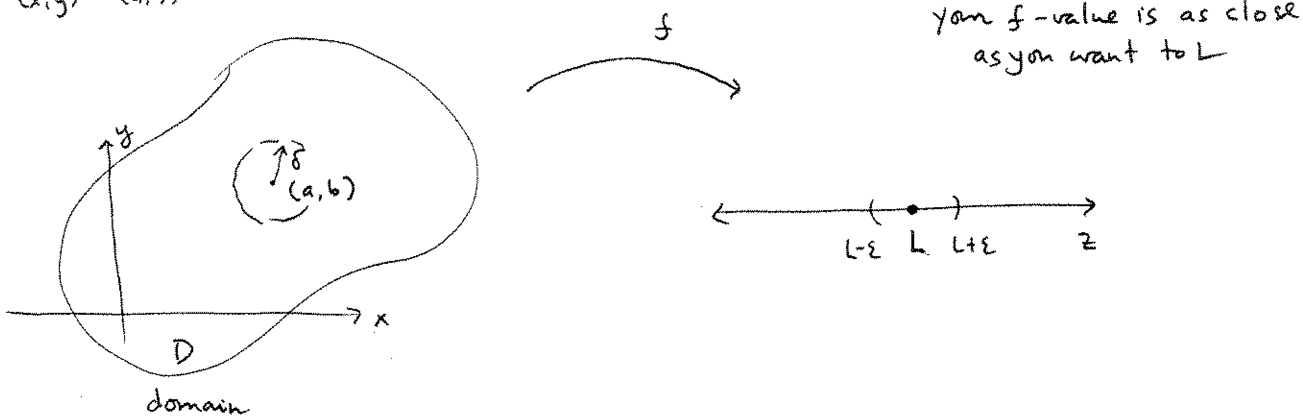
generalize what you did in Calc.

$f: D \to \mathbb{R}$, $D \subset \mathbb{R}^n$

$f(x,y)$
$f(x,y,z)$

etc.

$\lim_{(x,y)\to(a,b)} f(x,y) = L$ means: Should mean: if you are close enough to (a,b) in the domain, then your f-value is as close as you want to L



precise def: for all $\varepsilon > 0$ there exists a $\delta > 0$ so that

$\| \langle x,y\rangle - \langle a,b\rangle \| < \delta$
$\Rightarrow |f(x,y) - L| < \varepsilon$

What would this mean in terms of the graph $z = f(x,y)$?

Examples Find

$\lim_{(x,y)\to(2,0)} 3x$

$\lim_{(x,y)\to(2,0)} 4y$

$\lim_{(x,y)\to(2,0)} e^{3x}(4y^2+1)$

A function $f(x,y)$ is <u>continuous</u> at (a,b) iff

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b).$$

We have the same limit theorems as in 1-variable Calc:

If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ then $f+g \to L+M$

$\lim_{(x,y)\to(a,b)} g(x,y) = M$ $\quad fg \to LM$

$f/g \to L/M$ (as long as $M \neq 0$).

So if f, g are cont (at (a,b)) then so are fg, $f+g$, f/g (if $g(a,b) \neq 0$).

Also, if $f(x,y)$ is cont at (a,b)
and g is cont at $f(a,b)$
then $g\circ f$ is cont at (a,b)

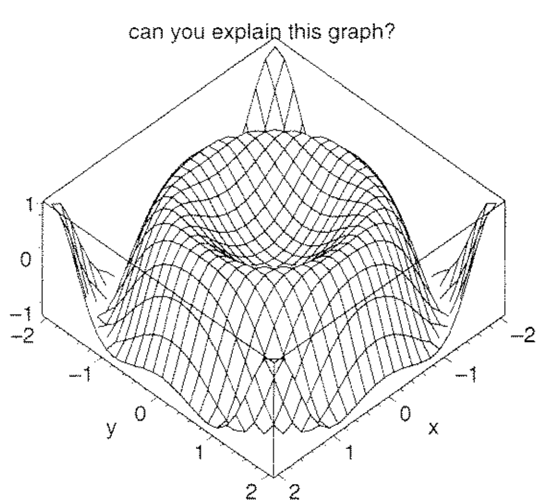
etc.

Analogous for functions of (x,y,z), (x,y,z,w) etc.

If a function is continuous, you can get the limit by "plugging in"

1. $\lim_{(x,y)\to(0,0)} \sin(x^2+y^2)$

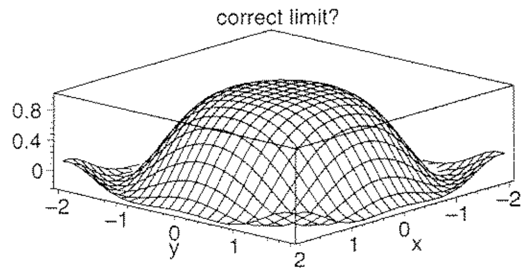
$z = \sin(x^2+y^2)$



2) $\lim_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{(x^2+y^2)}$

$z = \dfrac{\sin(x^2+y^2)}{(x^2+y^2)}$

$\lim_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2} = ?$

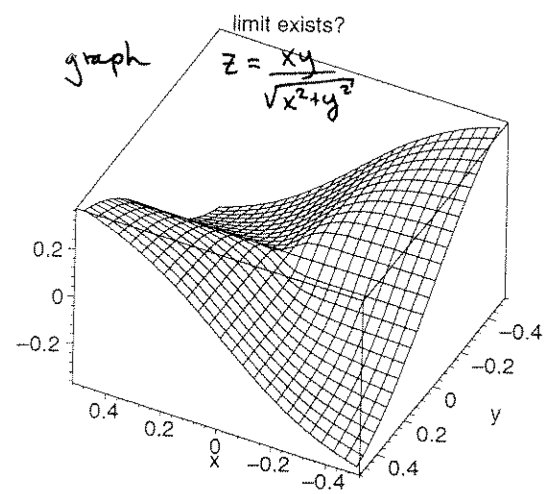


2b) Could you define $f(0,0)$ to make the function above continuous at $(0,0)$?

3) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$

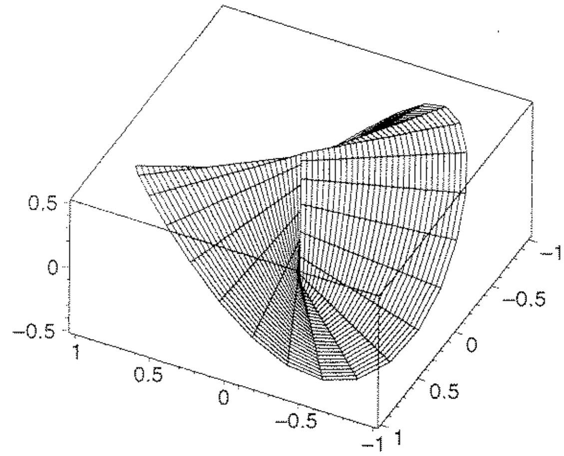
try polar coords!

graph $z = \dfrac{xy}{\sqrt{x^2+y^2}}$



(4) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

try polar coords!
or look along lines thru origin

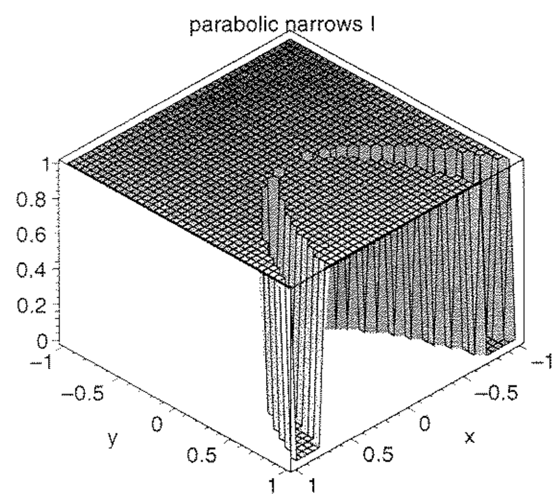


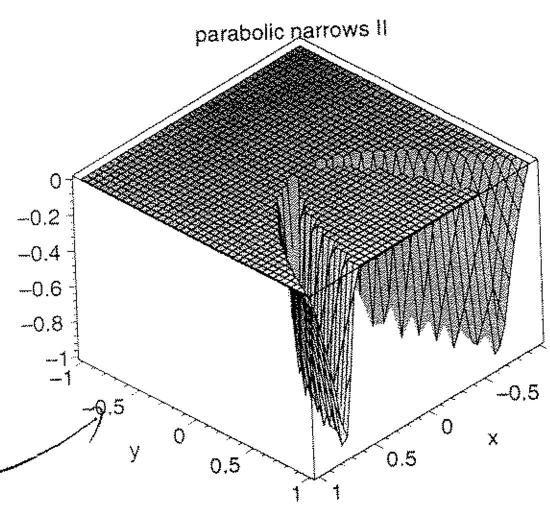
(5) $f(x,y) = \begin{cases} 1 & y \geq 2x^2 \\ & \text{or } y \leq x^2 \\ 0 & \text{if } x^2 < y < 2x^2 \end{cases}$

does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

(notice if you approach along any line thru the origin, the limit of f is 1)

at what points is this f discontinuous?





only discont. at origin!