Math 2210-3
Monday Feb 22

§ 11.2 Partial derivatives

$$f : D \to \mathbb{R}$$
$$D \subset \mathbb{R}^n$$

$f(x,y)$
$f(x,y,z)$
$f(x,y,z,t)$ etc.

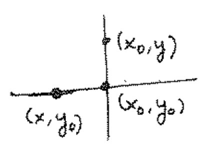
partial deriv. of $f$ with respect to $x$, at $(x_0, y_0)$:

$$f_x(x_0,y_0) := \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x, y_0) - f(x_0,y_0)}{\Delta x}.$$

measures rate of change of $f$ in $x$-direction.

In fact, it's just a 1210 derivative

$g'(x_0)$ ; $g(x) = f(x, y_0)$ ($y_0$ const, $x$ variable)




partial deriv. of $f$ w.r.t. $y$, at $(x_0,y_0)$:

$$f_y(x_0,y_0) := \lim_{\Delta y \to 0} \frac{f(x_0, y_0+\Delta y) - f(x_0,y_0)}{\Delta y}$$

measures rate of change of $f$ in $y$-dir.
Is just a 1210 deriv

$h'(y_0)$; $h(y) = f(x_0, y)$ ($y$ variable.)

So, partial derivs are easy to compute!

$f(x,y) = e^{2x}\cos 3y$

$f_x =$

$f_y =$

$f_x(0,\pi) =$

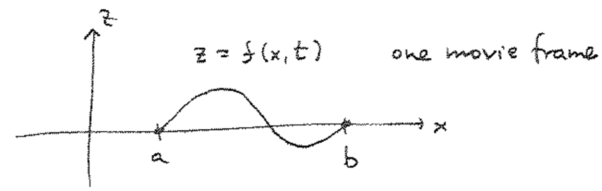
$f_y(0,\pi) =$

What should be the definition of $f_z(x_0,y_0,z_0)$?

# Geometric and Physical interpretations

- Suppose $f(x,t)$ is the vertical displacement of a vibrating string $(a \le x \le b)$ at time $t$.

  What do $f_x(x_0,t_0)$, $f_t(x_0,t_0)$ represent physically?

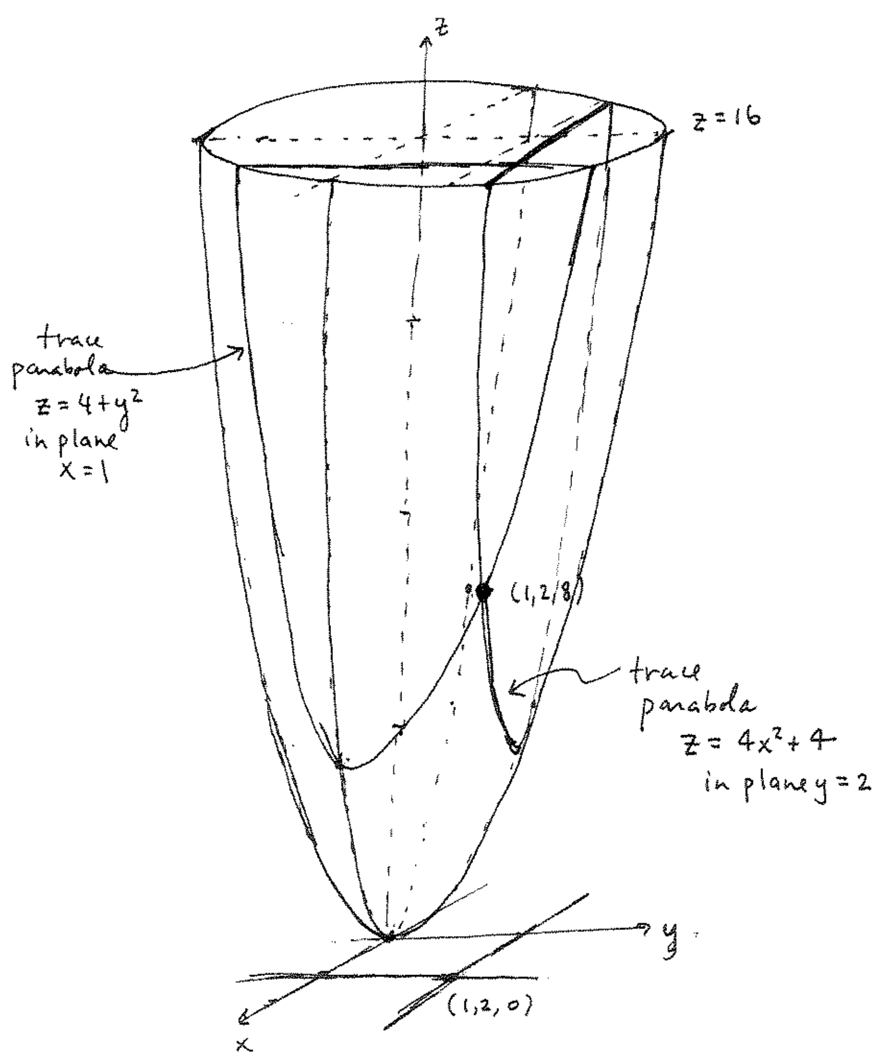



- Slopes of trace curves:

function $f(x,y) = 4x^2+y^2$

graph $z = 4x^2+y^2$

- Interpret $f_x(1,2)$ and $f_y(1,2)$ as slopes of trace curves.
- Can you find tangent vectors to these curves at $(1,2,8)$?
- Could you find the tangent plane to the paraboloid at $(1,2,8)$?

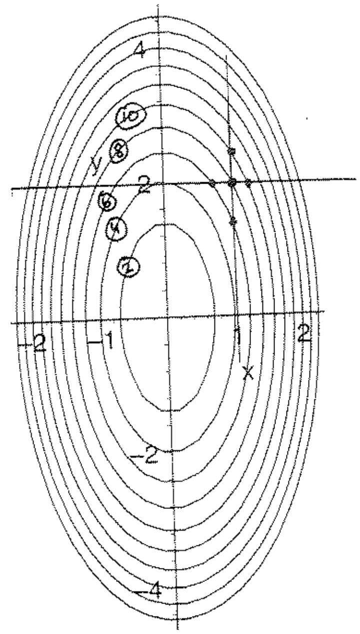


- Re-estimate

$f_x(1,2)$
$f_y(1,2)$ for $f(x,y) = 4x^2 + y^2$

from this "contour map" of the level curves of $f$, using an index card ruler.

level curves of f, in the domain ~ $f=2, f=4, f=6$ etc.




At $(1,2)$,
I get $f_x \approx \frac{\Delta f}{\Delta x} \approx \frac{4}{.5} = 8$

$f_y \approx \frac{\Delta f}{\Delta y} \approx \frac{4}{1.1} \approx 4$

How about you?

Other notation for partial derivatives, and higher order partial derivs

$$f_x = \frac{\partial f}{\partial x} = D_x f$$

$$f_y = \frac{\partial f}{\partial y} = D_y f$$

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = D_y D_x f$$

$$f_{xxy} = ((f_x)_x)_y \quad \text{etc.}$$

<u>Example</u>: Compute all $1^{st}$ and $2^{nd}$ order partial derivatives for $f(x,y,z) = z\sin(x/y) + xy^2z^3$.

What do you notice?