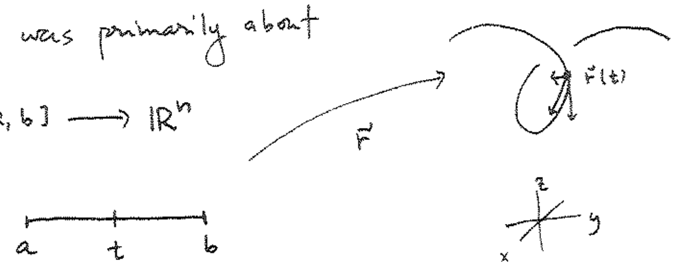
Math 2210-3
Friday Feb. 19

HW for Wed. Feb. 24
12.1 1, 9, 10, 11, 17, 18, 23, 24
25, 27, 33, 35, 41
12.2 1, 13, 17, 25, 27, 39, 47

Chapter 11 was primarily about

$\vec{r}: [a,b] \longrightarrow \mathbb{R}^n$



domain in $\mathbb{R}$ range in $\mathbb{R}^n$

Chapter 12 is primarily about the "reverse" situation

$$f: \underset{\mathbb{R}^n}{\overset{}{D \atop \cap}} \longrightarrow \mathbb{R}$$

(D = domain)

real examples:

$f(x,y)$ = altitude above sea level (on earth) at longitude $= x$, latitude $= y$

$T(x,y,t)$ = temperature on earth at time t

$T(x,y,z,t)$ = same, but also fcn of altitude.

hybrids

$\vec{E}(x,y,z,t)$ electric field at time t

$\vec{B}$ magnetic

$\vec{F}(x,y,z,t)$ any force field

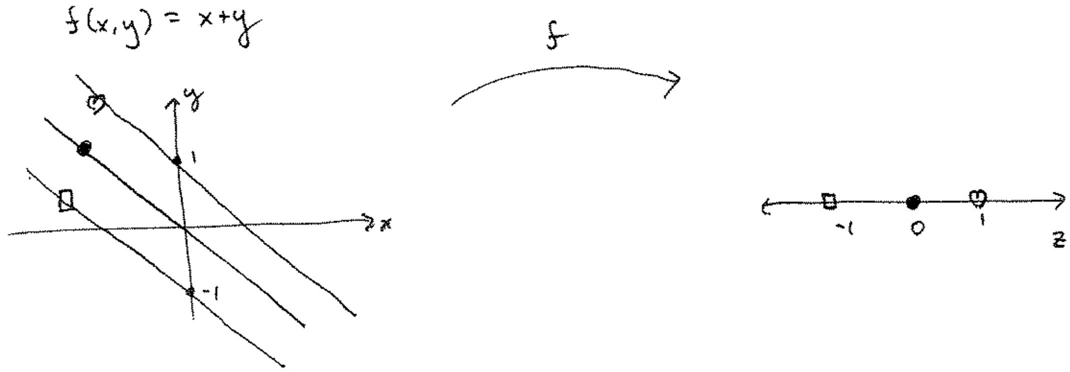
Other courses you will take soon where these ideas come up

$\vec{r}(t)$: Ordinary differential eqtns, Math 2250 (or 2280)

$f(x_1, \ldots x_n)$: Partial differential eqtns, Math 3150 (or 2280)

<u>Example</u>

$f(x,y) = x+y$

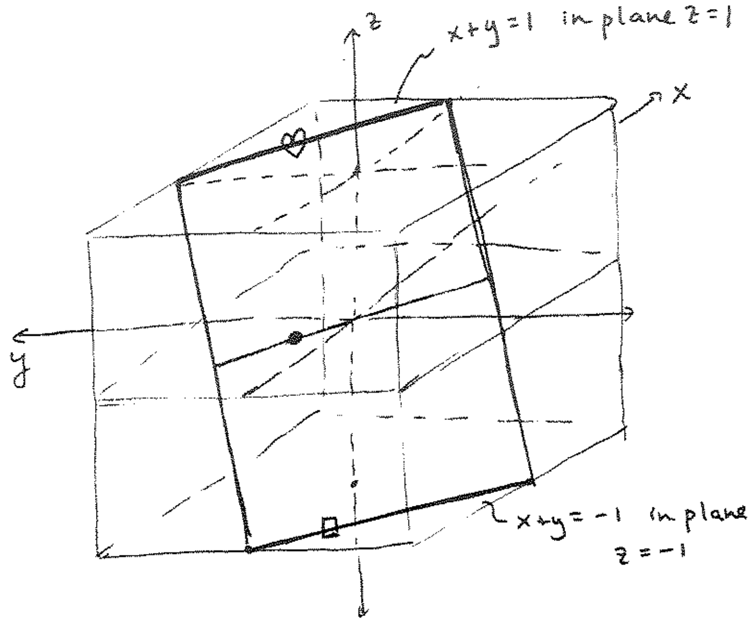


"level curves"

If the domain D is in $\mathbb{R}^2$
then you visualize the function f
by drawing the graph $z = f(x,y)$ in $\mathbb{R}^3$

Can you draw the graph

$$z = x+y \,?$$



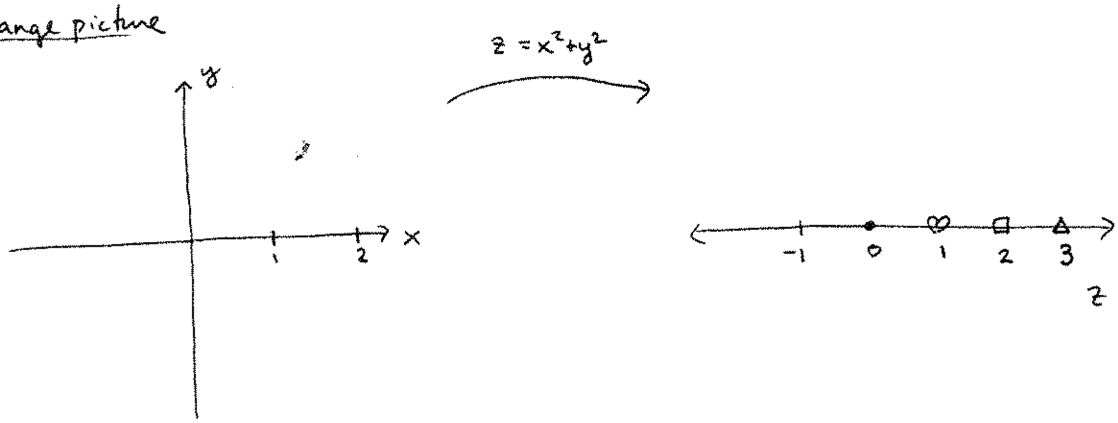
Intersections of a graph with
horizontal planes ($z = \text{const}$)
are called <u>contours</u> (on a hike you might follow a contour)

The projections of contours of a graph $z = f(x,y)$
back to the
domain in the x-y plane are
called <u>level curves</u>

(and then, just to confuse you, a picture in
the domain containing sample level curves is called a <u>contour map</u>!

<u>Example</u> $g(x,y) = x^2+y^2$

<u>domain-~~range~~ picture</u>



<u>graph $z = x^2+y^2$ in $\mathbb{R}^3$</u>

example

$f: \mathbb{R}^3 \to \mathbb{R}$

$f(x,y,z) = x^2+y^2-z^2$

domain picture for ♡, •, □:

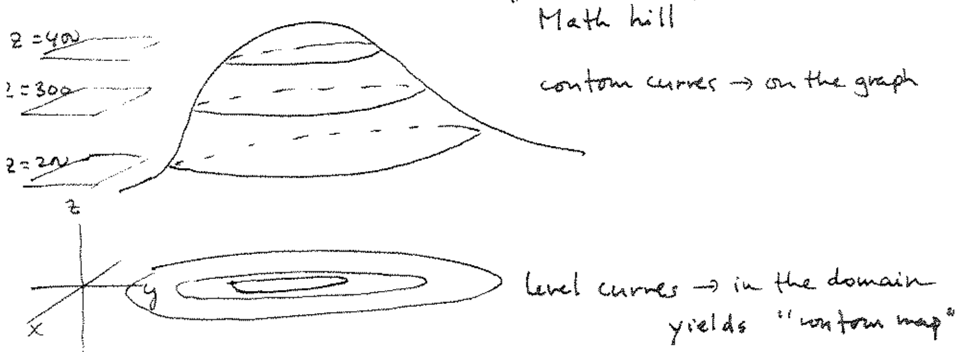
range

graph?

<u>Def</u>: If $f : D \to \mathbb{R}$, $D \subset \mathbb{R}^2$

then the intersection of of a horizontal plane $z = c$
with the graph $z = f(x,y)$ is called a <u>contour curve</u>

the set of (x,y) in the domain of f for which $f(x,y) = c$ is called a <u>level curve</u>
thus the level curve is the projection of the contour curve back to the
x-y domain.



anyone who understands topo maps has used level curves of elevation to
understand the 3-d graph of the elevation function → see also p. 620 - 621 pictures

Can you draw the level curve (contour map)
picture for

$$f(x,y) = x^2 - y^2$$

and the graph $z = x^2 - y^2$, with contours?

<u>Def</u>: If $f : D \to \mathbb{R}$, $D \subset \mathbb{R}^3$, then the set of (x,y,z) in $\mathbb{R}^3$ for which $f(x,y,z) = c$ (constant)
is called a <u>level surface</u> of f. For example, return to page 3.

Something to think about

We've talked about explicit (parametric) and implicit representations for lines, planes, curves, surfaces.

in the explicit case we are saying the object is the range of a certain function

in the implicit case we are saying the object is a level curve, level surface, or intersection of level surfaces in the domain of a function.

Also, have a leisurely look at the nice 3-d graphs and 2-d level curve pictures, p. 620-621 text.
(but careful)