Math 2210-3 / Wed Feb 10

- Discuss §11.8 : quadric surfaces
  (we'll go through the examples in Monday's notes)

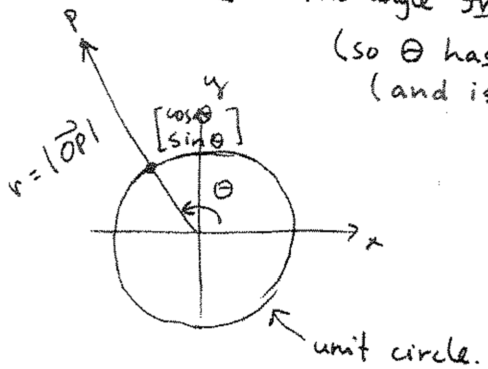
with any remaining time we'll discuss the beginning of §11.9 cylindrical and spherical coords in $\mathbb{R}^3$

First, recall <u>polar coordinates</u> (r,θ) in $\mathbb{R}^2$:

For $P=(x,y) \in \mathbb{R}^2$

$r := \sqrt{x^2+y^2} = \text{dist}(P,0)$

$\theta :=$ the angle <u>from</u> the x-axis to $\overrightarrow{OP}$
(so θ has a sign)
(and is uniquely defined up to a multiple of 2π



$r = \sqrt{x^2+y^2}$

$\theta = \begin{cases} \arctan y/x & \text{quadrant I, IV} \quad (-\pi/2 \le \theta \le \pi/2) \\ \arccos \frac{x}{r} & \text{I, II} \quad (0 \le \theta \le \pi) \\ \arcsin \frac{y}{r} & \text{I, IV} \quad (-\pi/2 \le \theta \le \pi/2) \\ \arctan y/x + \pi & \text{quadrant III} \quad (\pi \le \theta \le 3\pi/2. \end{cases}$

etc.

in reverse direction is easier to express :

$\langle x, y\rangle = \langle r\cos\theta, r\sin\theta\rangle$
$= r\langle \cos\theta, \sin\theta\rangle$

<u>amples</u>:

if $r=5$, $\theta = \pi/3$ $\begin{bmatrix}x\\y\end{bmatrix} =$

if $\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-10\\10\end{bmatrix}$, $\begin{bmatrix}r\\\theta\end{bmatrix} =$

draw the curve implicitly defined by $r=3$

draw the curve implicitly defined by $\theta = \frac{8}{3}\pi$ ($\& r>0$).

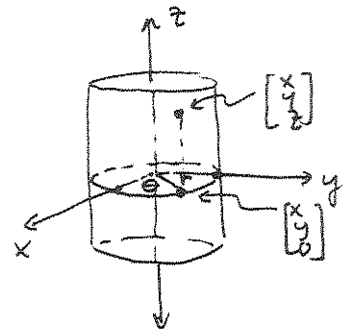
Why are these coords called "polar coords"?

HW for Wed Feb 17
(also exam 1 day!)

11.8 2, 5, 7, 8, 9, 13, 14, 29
11.9 2a, 3a, 4a, 5a, 6a, 7, 8, 9, 13 15, 17, 23, 27, 35, 36

I'll post review notes & practice exams on Thursday. Do you want an exam review session during regular Tuesday session, or on Monday (holiday)?

Exam will cover Chapter 11.



$\mathbb{R}^3$: cylindrical coordinates:

use $\begin{bmatrix} r \\ \theta \end{bmatrix}$ in the x-y plane, keep z

i.e. $\begin{bmatrix} r \\ \theta \\ z \end{bmatrix}$ } polar coords (for r, θ), } z (for z)

examples: What are the $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ("rectangular") coords of a point with cylindrical coords $\begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$?

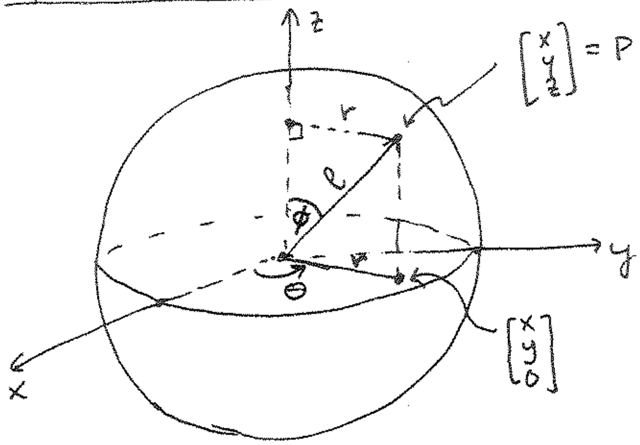
if $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$ what are $\begin{bmatrix} r \\ \theta \\ z \end{bmatrix}$?

Sketch the surface implicitly defined by $r=3$. Why are these coords called cylindrical coords?

Sketch the surface implicitly defined by $\theta = \pi/4$ (& $r>0$)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ z \end{bmatrix}$$

spherical coordinates



$\rho := \sqrt{x^2+y^2+z^2}$

$\phi :=$ angle between pos z axis ($\hat{k}$) and $\vec{OP}$

$0 \leq \phi \leq \pi$ $\rightarrow$ so $\phi = \arccos\left(\frac{z}{\rho}\right)$

$\theta :=$ polar coord angle of projection $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

so (see triangles in picture)

$z = \rho\cos\phi$

$r = \rho\sin\phi$ (polar r)

so $x = r\cos\theta$
$= \rho\sin\phi\cos\theta$

$y = r\sin\theta$
$= \rho\sin\phi\sin\theta$

examples

if $\begin{bmatrix} \rho \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{5}{6}\pi \\ \frac{1}{3}\pi \end{bmatrix}$, What is $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$?

if $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, what is $\begin{bmatrix} \rho \\ \theta \\ \phi \end{bmatrix}$?

sketch the surfaces implicitly defined by

(1) $\rho = 3$;

(2) $\theta = \pi/4$;

(3) $\phi = \pi/3$;

$$\boxed{\begin{aligned} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{aligned}}$$

other examples:

What surface is implicitly defined
by the spherical coord eqtn

$$\rho = 2\cos\phi \,?$$

(hint: convert to rectangular coords)

How would you find the great-circle distance between 2 points
on Earth, if you knew their latitude & longitude?

Example 7 p. 611-612 &HW