Math 2210-3
Monday Feb. 1.

- Recall derivatives (& 2nd derivatives) for vector-valued fcns $\vec{r}(t)$. There is a list of differentiation rules we need to understand & be able to use, on page 5 of Friday's notes. Also an example of antidifferentiation
- Then §11.6 : Lines & tangent lines

Recall, we've already talked about parametric lines in $\mathbb{R}^2$ & $\mathbb{R}^3$, with position vectors $\vec{r}(t) = \vec{a} + t\vec{b}$

$\vec{a}$: $\vec{r}(0)$

$\vec{b}$: direction of line. In fact $\vec{r}'(t) = \vec{b}$ so $\vec{b}$ is the constant velocity vector

Example 1. In $\mathbb{R}^2$ we've gone back & forth between parametric form and implicit eqtns for lines:

e.g. express the line thru $(1,1)$ and $(3,-4)$ parametrically
& in slope-pt form

If $\vec{r}(t) = \vec{a} + t\vec{b}$ is a parametric line in $\mathbb{R}^3$, and you solve for t in each of the equations

$$\begin{aligned} x &= a_1 + t b_1 \\ y &= a_2 + t b_2 \\ z &= a_3 + t b_3 \end{aligned}$$

you get $t = \boxed{\dfrac{x-a_1}{b_1} = \dfrac{y-a_2}{b_2} = \dfrac{z-a_3}{b_3}}$ (assuming $b_1, b_2, b_3 \neq 0$)

this is called the "symmetric" equations of the line. You're really giving implicit equations for the line thru (a_1, a_2, a_3) as an intersection of planes. Notice how if you are given such equations, you can recover a parametric description

Example 2: Find a parametric curve description for the line with symmetric form

$$\frac{x-2}{3} = \frac{y-1}{4} = \frac{z+3}{2}$$

How many such parametric curve descriptions are there?

Example 3

Find symmetric and parametric equations for the line of intersection between the two planes

$$x - y + 2z = 5$$
$$2x + y + z = 1$$

Lots of ways to do this problem!

- solve simultaneously
- use cross product

<u>Definition</u>: The <u>tangent line</u> to a parametric curve with position vector $\vec{r}(t)$, $(a \le t \le b)$
at "time" t_0, is the line thru $\vec{r}(t_0)$, with direction $\vec{r}\,'(t_0)$.
(so, using a parameter τ, you could describe this line parametrically by $\vec{L}(\tau) = \underbrace{\vec{r}(t_0)}_{\vec{a}} + \underbrace{\vec{r}(t_0)}_{\vec{b}}\tau$)

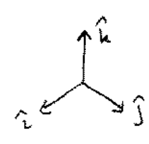
<u>Example 4</u> $\vec{r}(t) = \langle 2\cos t, 2\sin t, 2-2\sin t \rangle$ ⟵⟶ How did I find this parameterization?
$0 \le t \le 2\pi$
parameterizes the intersection of the cylinder $x^2+y^2=4$
and the plane $y+z=2$

a) Plot $\vec{r}(0)$ (as a point)
$\vec{r}\,'(0)$ (as a vector starting at $\vec{r}(0)$)

b) Find a parametric representation of the tangent line thru $\vec{r}(0)$

c) symmetric form.

d) Find eqtn of plane thru $\vec{r}(0)$ perpendicular to the tangent line



intersection of cylinder and skew plane